Community College Study of Mathematical Concepts and Skills Retention in Elementary Algebra: The Role of Distributed Practice and Problem-Centered Learning

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Background and Significance

One of the greatest challenges in teaching college mathematics at the remedial level is that students do not remember what they have learned from one week to the next. There are many reasons for this—some academic (curriculum, teaching styles) and some personal (lack of preparation, poor study habits, language barriers, test and math anxiety). At Bronx Community College approximately 70% of the incoming students are placed into pre-algebra or elementary algebra. The passing rate in the elementary algebra course at the college is approximately 50%. Since most students who attend one of CUNY’s community colleges are placed into a developmental mathematics course, any improvement in teaching that leads to better understanding and retention may have an impact on passing rates and ultimately on graduation rates.

Related Research

Distributed practice involves spreading the practice problems over time as opposed doing all the problems in one session. In a typical mathematics class, after covering a topic, such as how to add signed numbers, students are given a number of examples to do in class and a homework assignment on that topic. This type of learning/teaching model is referred to as massing or overlearning (Rohrer and Taylor, 2006). In the distributed practice model, the same practice problems are spread across several assignments.

An experiment conducted by Rohrer and Taylor (2006) was designed to determine the effect of distributed practice compared to overlearning. They found that long-term retention was much greater in the group of students who were in the distributed practice group, as compared to students who engaged in overlearning.

Schroeder and Lester (1989) describe three ways that teachers used problem-solving: teaching about it; teaching for it; and teaching via it. In their opinion, the last is consistent with the recommendations in the National Council of Teachers of Mathematics’ (NCTM) Standards of 1987 that “(1) mathematics concepts and skills be learned in the context of solving problems; (2) the development of higher-level thinking processes be fostered through problem-solving experiences; and (3) mathematics instruction take place in an inquiry-oriented, problem-solving atmosphere (NCTM 1987).”

Objectives of the Project

The objective of this project was to improve students’ understanding and retention of mathematical concepts and skills in Elementary Algebra using distributed practice in a problem-
centered context. The usual approach at Bronx Community College is to teach new material in a context-free framework and to assign homework exercises based on the topic covered (a massed or overlearning approach).

There were two experimental treatments. One revised the way that homework exercises were assigned. The exercises were spread over the entire semester rather than being assigned all at once. The second added more real-life problems to provide motivation and rationale for the material being taught.

The problems used to improve students’ problem-solving skills were drawn from topics related to students’ majors (business, engineering, health and human services, radiologic technology and the sciences) as well as from areas important to their lives (personal finance, citizenship, statistics). These types of problems were found in textbooks, in the literature, and from sources such as the Consortium for Mathematics and its Applications (COMAP), or they were developed by the project personnel. The problems were chosen to reflect authentic, real-world situations unlike those generally presented at the end of drill-and-practice exercises in most textbooks.

The hypotheses to be tested were: (1) students in the experimental sections will have a higher passing rate on the departmental final exam; (2) problem-centered learning will enhance students’ problem-solving ability; (3) students in the experimental sections will have a higher attendance rate than students in the control sections.

**Design and Methodology**

The experimental design was “quasi-experimental” in nature. In the school setting, it was not possible to assign students randomly to control or treatment course sections. Nonetheless it was still possible to compare the outcomes of students in one group with those of students in a similar group. According to Moore (2008), “Quasi-experimental studies can inform discussions of cause and effect, but, unlike true experiments, they cannot definitively establish this link.” In a quasi-experimental study, it is not possible to control all the variables that would establish the cause-effect relationship; however, it is still possible to control a certain number of them to relate positive outcomes to experimental treatment.

This experiment took place over the course of three semesters: Spring 2010, Fall 2010 and Spring 2011. In Spring 2010 the PI and Co-PI each taught one section of Elementary Algebra using the departmental syllabus as written, assigning the homework exercises as specified on the syllabus using the strategy of assigning homework—overlearning. In Fall 2010 the same instructors each taught a section using distributed practice with the practice exercises spread over the semester. In Spring 2011, they each taught a section using a problem-centered approach with distributed practice. The problems were designed to motivate the students and provide an incentive for learning the material needed to solve the problems. The class sections met at the same time of day and the same days of the week each semester. The unit tests given were similar for both the control sections and the experimental sections. All students took the department’s final examination.
According to Moore (2008), it is still possible to analyze the results in a meaningful way to “control for measured and unmeasured variables.” A multiple regression test can be used to control for confounding factors or analysis of variance can be used to determine whether differences exist between the three groups. However, this study employs t-tests and simple mean comparisons, because covariate data were not available, and the experimental design controls for instructor, types of students, and treatment, insofar as possible. For example, the two instructors each taught the control sections and the two treatment sections. Sections were taught at the same time of day, which may help minimize differences in student demographics between class sections. Introducing distributed practice in one semester and adding a problem-centered approach the next semester enabled us to look at the effects separately.

The investigators designed a syllabus with practice exercises spread across multiple assignments to use in the two experimental treatments. For the third semester, when the problem-centered approach was used, the investigators developed a set of problem booklets that intersperse new and previously-taught material. The booklets provided an opportunity for students to work in groups to develop strategies to solve a variety of problems that lead to a discussion of the underlying mathematics. After each unit of instruction, students in the control and experimental sections were given a test on new and previously-taught material. Student retention of concepts and skills was analyzed using the departmental final exam and the results were compared for the experimental and control sections.

In order to maintain student confidentiality, no identifiers were used in the reporting of data. All students test records were stored in a locked filing cabinet in the PI’s office. Before any data were collected, a representative of the Office of Institutional Research came to each section, explained the nature of the experiment and handed out a statement describing the experiment to allow each student to give or withhold consent. These forms were kept in the Office of Institutional Research until the final exams were graded and final grades assigned. This ensured that students didn’t feel coerced to participate, that they would be graded fairly and impartially, and that their results would only be used if they consented to participate.

**Hypothesis 1: Final exam scores will increase**

We analyzed the first hypothesis using a right-tailed statistical test of the null hypothesis based on the following data:

<table>
<thead>
<tr>
<th></th>
<th>Spring 2010</th>
<th>Fall 2010</th>
<th>Spring 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>32</td>
<td>35</td>
<td>41</td>
</tr>
<tr>
<td>(\bar{x})</td>
<td>57.09</td>
<td>64.60</td>
<td>64.20</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>24.47</td>
<td>21.62</td>
<td>20.82</td>
</tr>
<tr>
<td>(Z)</td>
<td>2.055</td>
<td>2.19</td>
<td></td>
</tr>
</tbody>
</table>

where \(N\) is the number of students; \(\bar{x}\) is the mean score on the final examination, \(\sigma\) is the standard deviation of the population and \(z\) is the score calculated using the formula \[ z = \frac{\bar{x} - \mu}{\sigma/\sqrt{N}}. \] We formulated the null and alternative hypotheses as follows:
$H_0: \mu = \text{or} < 57.09$ The average score on the final exam in the experimental sections will be less than or the same as in the control sections.

$H_1: \mu > 57.09$ The average score on the final exam in the experimental sections will be greater than the average score in the control section.

Based on the results we can reject the null hypothesis at the 5% confidence level for the experimental sections. Our conclusion is that the interventions were successful in increasing passing rates on the final exam.

**Hypothesis 2: Students in the problem-centered sections (Spring 2011) will be better problem solvers**

The problems on the final examination in Spring 2010 were considerably harder than those in subsequent semesters. Consequently, we could not do any analysis comparing the control and experimental sections. However, we did have data from two other Spring 2011 day sections that had the same type of word problems, and thus may serve as a comparison group. These problems were covered in all sections of the course. As can be seen in the following Figure, students in the experimental sections performed, on average, better than those in the other two sections.

![Average Scores on Word Problems](image)

When we looked at the average scores, on a scale from 0 to 5, for the experimental and comparison sections we observed that students in the experimental sections performed considerably better than those in the comparison sections on the following word problems:

**Rectangle Problem**: The length of a rectangle is four less than two times the width. The perimeter of the rectangle is 64 inches. Find the length and the width of the rectangle.

**Linear Equation Problem**: Find the slope and y-intercept of the line passing through the points $(2; 7)$ and $(7; -3)$. Then, write an equation for the line.
**Consecutive Integer Problem:** Find three consecutive integers such that the sum of the first two numbers is fourteen more than the third. Write and use an algebraic equation to solve this problem.

**Right Triangle Problem:** A right triangle has a hypotenuse measuring 10 in and one leg measuring 4 in. Find the length of the third side. Express your answer in simplest radical form.

Except for the Linear Equation Problem, these distributions are like inverted bell curves, known as “well curves.” They are bimodal and cannot be analyzed by any of the standard statistical methods.

**Hypothesis 3:** Retention of students in class will increase and a higher percentage of students will pass the final exam.
We examined the data in two ways. First we looked at the number of students who completed the course compared with the number of students who were enrolled at the beginning of the semester. Then we compared that number with the number of students who passed the final exam.

![Retention Rates and Final Exam Passing Rates](image)

Although there is essentially no difference in the retention rate across the three sections, there is a steady increase in the percentage of students who passed the final exam. This could account for the significant difference found between the average scores on the final exam between the experimental and control sections.

**Conclusions**

In summary, this study found evidence to support all three of the hypotheses. It appears that both experimental treatments, distributed practice and problem-centered learning, had a significant impact on learning and retention of concepts and skills as evidenced in the final examination results. Problem-centered learning contributed to students’ increased ability to solve word problems. Finally, students in the experimental section had higher attendance rates, as demonstrated by their higher rate of course completion.

When considering these results, there are several limitations that must be considered. For example, the analyses for hypotheses two and three employ descriptive statistics rather than tests of statistical significance. These analyses could have also benefitted from the inclusion of control variables to account for student background characteristics that may have differed across class sections and semesters. Finally, there is the possibility that differences across cohorts, for example, in the content of the final exam or in instructors, may have biased results. Nevertheless, this study provides preliminary evidence that distributed practice and problem-centered learning hold promise for students in remedial math courses at CUNY.
Next steps

These results will be presented at the meeting of the department. The hope is that the faculty will adopt the syllabus designed for distributed practice and will use the problems developed for the problem-centered treatment.

Professor Bates will teach one section of Elementary Algebra in Fall 2011 using distributed practice and problem-centered learning. She will also try to obtain final examination results and withdrawal rates for each of the day sections. She plans to follow the students in the control and experimental sections to determine whether distributed practice contributes to long term retention of concepts and skills. She will try to determine whether there are statistical tests of significance for inverted bell curve distributions. (Professor Forman retired from BCC so will not participate in follow-up activities.)

In Spring 2012, she will analyze the results and write a paper for publication in an appropriate journal such as the *MathAMATYC Educator*. Additional funding to enlarge the project will be sought from various agencies such as NSF’s TUES program or from the US Department of Education’s Hispanic Serving Institutions—Science, Technology, Engineering, or Mathematics program.

In the Fall of 2012, Dr. Bates will follow up with students who were in the distributed practice group to gather data on their retention of the course material. This will allow for a test of the hypothesis that distributed practice increases long-term retention.

Acknowledgements

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References

COMAP: Mathematics Instructional Resources for Innovative Educators
(http://www.comap.com)

