Improving Undergraduate Mathematics Learning

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**Introduction**

In Spring 2009, CUNY’s Office of Academic Affairs launched a grant program, Improving Undergraduate Mathematics Learning (IML), to support studies of undergraduate mathematics instruction by CUNY faculty members. From its inception, IML has aimed to identify promising practices and programs in CUNY’s undergraduate mathematics classrooms and to address demonstrated gaps in math knowledge among CUNY’s undergraduate population by supporting evidence-based research projects.

The program’s request for proposals (RFP) announced the availability of grants. Faculty were encouraged to assess innovations that they had devised or had adapted from other institutions. In response to the RFP, 33 letters of intent were received. Respondents were encouraged to attend a research design workshop in September 2009 to refine their proposals.

In all, 26 full proposals were submitted the following month, and the IML review panel—three senior-level mathematics faculty and three Central Office administrators with research expertise—relied on a blind review process to assess the competitive pool of applicants. In November, ten faculty teams were chosen to receive grants ranging from $32,000 to $86,000, depending on the scope and extent of the proposal. The funded investigators included 27 faculty members from eight different CUNY campuses.

The IML projects explored topics including small-group tutoring, collaborative learning, student-centered learning, adaptive syllabi, study skills and time management training, peer coaching, problem-centered learning, distributed practice, online homework programs (or Internet-based homework programs), virtual manipulatives, and modular workshops. Faculty investigators measured success in terms of successful course completion, student retention rates, increased test scores, and measurable improvement in students’ confidence and attitude towards mathematics. Six of the projects investigated performance in remedial mathematics classes, and the other four projects investigated performance in credit-bearing mathematics classes.

More specifically, three studies investigated the role of peers in the learning process, either in groups completing collaborative assignments, or as peer tutors and instructors. Betne, Dedlovskaya, Toce, Wang, and Zaritsky (LaGuardia Community College) assessed whether peer tutoring increases student study time and course performance. Khazanov, George, and MacCarthy (Borough of Manhattan Community College) paired peer coaches with students deemed to be at risk of failure. The mentors provided study skills training and tools to manage test anxiety. Wladis and Morgulis (Borough of Manhattan Community College) tested the hypothesis that a change in the instructional format from a traditional lecture-based class to one emphasizing student collaboration increases student success. Results of all of these studies suggest that peer learning is a promising approach. Students who received peer tutoring services spent substantially more time studying. Peer coaching and collaborative work in stable peer groups were also linked to higher rates of course completion and greater likelihood of earning a passing course grade.
Three of the IML projects explored promising new teaching strategies. Bates and Forman (Bronx Community College) employed the principle of distributed practice, by spreading homework exercises on one topic across several assignments rather than assigning them all at once. Guy, Cornick, Holt, and Russell (Queensborough Community College) tested a modular workshop class format that emphasized student problem solving and differentiated instruction, versus a traditional lecture format for students in remedial arithmetic. Kingan, Clement, and Hu (Brooklyn College) tested the hypothesis that a preparatory workshop offered at the beginning of the semester could help fill gaps in student math knowledge and improve course performance. Findings demonstrated that the modular workshop format and preparatory workshops were associated with improvement in performance. Distributed practice homework assignments also appeared to have positive effects on student retention of class material and course performance.

Finally, three studies examined new instructional technology. Cunningham and Dias (Hostos Community College) tested the effect of online homework versus pencil-and-paper homework, in addition to the effect of increasing the number of tutors available to provide homework help. Similarly, Grossman, Ocken, and Schonfeld (City College) tested the effect of internet-based homework and content resources versus the pencil-and-paper approach in a pre-calculus course. Menil and Fuchs (Hostos Community College/Bronx Community College) investigated whether teaching students through instructional technology that allowed them to manipulate virtual objects on a computer screen to solve math problems would result in better course performance in pre-algebra and algebra than traditional lecture-based instruction. These studies showed mixed results on measures of student performance. However, qualitative survey data indicate that students were enthusiastic about this new approach to instruction. Further research may be needed to determine the student subgroups and settings in which this popular new technology may be most effective.

The IML grant teams have recently submitted their final reports, which are available on the Math Matters website (www.cuny.edu/mathmatters). Further information on findings will be disseminated in conference presentations, CUNY publications, and peer-reviewed journals; many teams plan to seek additional funding to continue their research, and some teams have already begun to seek such funding.

The present publication provides a summary of each one of the IML projects.
Community College Study of Mathematical Concepts and Skills Retention in Elementary Algebra: The Role of Distributed Practice and Problem-Centered Learning

Professor Madelaine Bates, Department of Mathematics and Computer Science
Professor Susan Forman, Department of Mathematics and Computer Science
Bronx Community College
Background and Significance

One of the greatest challenges in teaching college mathematics at the remedial level is that students do not remember what they have learned from one week to the next. There are many reasons for this—some academic (curriculum, teaching styles) and some personal (lack of preparation, poor study habits, language barriers, test and math anxiety). At Bronx Community College approximately 70% of the incoming students are placed into pre-algebra or elementary algebra. The passing rate in the elementary algebra course at the college is approximately 50%. Since most students who attend one of CUNY’s community colleges are placed into a developmental mathematics course, any improvement in teaching that leads to better understanding and retention may have an impact on passing rates and ultimately on graduation rates.

Related Research

Distributed practice involves spreading the practice problems over time as opposed doing all the problems in one session. In a typical mathematics class, after covering a topic, such as how to add signed numbers, students are given a number of examples to do in class and a homework assignment on that topic. This type of learning/teaching model is referred to as massing or overlearning (Rohrer and Taylor, 2006). In the distributed practice model, the same practice problems are spread across several assignments.

An experiment conducted by Rohrer and Taylor (2006) was designed to determine the effect of distributed practice compared to overlearning. They found that long-term retention was much greater in the group of students who were in the distributed practice group, as compared to students who engaged in overlearning.

Schroeder and Lester (1989) describe three ways that teachers used problem-solving: teaching about it; teaching for it; and teaching via it. In their opinion, the last is consistent with the recommendations in the National Council of Teachers of Mathematics’ (NCTM) Standards of 1987 that “(1) mathematics concepts and skills be learned in the context of solving problems; (2) the development of higher-level thinking processes be fostered through problem-solving experiences; and (3) mathematics instruction take place in an inquiry-oriented, problem-solving atmosphere (NCTM 1987).”

Objectives of the Project

The objective of this project was to improve students’ understanding and retention of mathematical concepts and skills in Elementary Algebra using distributed practice in a problem-centered context. The usual approach at Bronx Community College is to teach new material in a context-free framework and to assign homework exercises based on the topic covered (a massed or overlearning approach).

There were two experimental treatments. One revised the way that homework exercises were assigned. The exercises were spread over the entire semester rather than being assigned all at once. The second added more real-life problems to provide motivation and rationale for the material being taught.

The problems used to improve students’ problem-solving skills were drawn from topics related to students’ majors (business, engineering, health and human services, radiologic technology and the sciences) as well as from areas important to their lives (personal finance, citizenship, statistics). These types of problems were found in textbooks, in the literature, and from sources such as the Consortium for Mathematics and its Applications (COMAP), or they were developed by the project personnel. The problems were chosen to reflect authentic, real-world situations unlike those...
generally presented at the end of drill-and-practice exercises in most textbooks.

The hypotheses to be tested were: (1) students in the experimental sections will have a higher passing rate on the departmental final exam; (2) problem-centered learning will enhance students' problem-solving ability; (3) students in the experimental sections will have a higher attendance rate than students in the control sections.

**Design and Methodology**

The experimental design was “quasi-experimental” in nature. In the school setting, it was not possible to assign students randomly to control or treatment course sections. Nonetheless it was still possible to compare the outcomes of students in one group with those of students in a similar group. According to Moore (2008), “Quasi-experimental studies can inform discussions of cause and effect, but, unlike true experiments, they cannot definitively establish this link.” In a quasi-experimental study, it is not possible to control all the variables that would establish the cause-effect relationship; however, it is still possible to control a certain number of them to relate positive outcomes to experimental treatment.

This experiment took place over the course of three semesters: Spring 2010, Fall 2010 and Spring 2011. In Spring 2010 the PI and Co-PI each taught one section of Elementary Algebra using the departmental syllabus as written, assigning the homework exercises as specified on the syllabus using the strategy of assigning homework—overlearning. In Fall 2010 the same instructors each taught a section using distributed practice with the practice exercises spread over the semester. In Spring 2011, they each taught a section using a problem-centered approach with distributed practice. The problems were designed to motivate the students and provide an incentive for learning the material needed to solve the problems. The class sections met at the same time of day and the same days of the week each semester. The unit tests given were similar for both the control sections and the experimental sections. All students took the department’s final examination.

According to Moore (2008), it is still possible to analyze the results in a meaningful way to “control for measured and unmeasured variables.” A multiple regression test can be used to control for confounding factors or analysis of variance can be used to determine whether differences exist between the three groups. However, this study employs t-tests and simple mean comparisons, because covariate data were not available, and the experimental design controls for instructor, types of students, and treatment, insofar as possible. For example, the two instructors each taught the control sections and the two treatment sections. Sections were taught at the same time of day, which may help minimize differences in student demographics between class sections. Introducing distributed practice in one semester and adding a problem-centered approach the next semester enabled us to look at the effects separately.

The investigators designed a syllabus with practice exercises spread across multiple assignments to use in the two experimental treatments. For the third semester, when the problem-centered approach was used, the investigators developed a set of problem booklets that intersperse new and previously-taught material. The booklets provided an opportunity for students to work in groups to develop strategies to solve a variety of problems that lead to a discussion of the underlying mathematics. After each unit of instruction, students in the control and experimental sections were given a test on new and previously-taught material. Student retention of concepts and skills was analyzed using the departmental final exam and the results were compared for the experimental and control sections.
In order to maintain student confidentiality, no identifiers were used in the reporting of data. All students test records were stored in a locked filing cabinet in the PI’s office. Before any data were collected, a representative of the Office of Institutional Research came to each section, explained the nature of the experiment and handed out a statement describing the experiment to allow each student to give or withhold consent. These forms were kept in the Office of Institutional Research until the final exams were graded and final grades assigned. This ensured that students didn’t feel coerced to participate, that they would be graded fairly and impartially, and that their results would only be used if they consented to participate.

Hypothesis 1: Final Exam Scores Will Increase

We analyzed the first hypothesis using a right-tailed statistical test of the null hypothesis based on the data shown in Table 1:

Table 1:

<table>
<thead>
<tr>
<th></th>
<th>Spring 2010</th>
<th>Fall 2010</th>
<th>Spring 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>32</td>
<td>35</td>
<td>41</td>
</tr>
<tr>
<td>( \bar{x} )</td>
<td>57.09</td>
<td>64.60</td>
<td>64.20</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>24.47</td>
<td>21.62</td>
<td>20.82</td>
</tr>
<tr>
<td>( z )</td>
<td>2.055</td>
<td></td>
<td>2.19</td>
</tr>
</tbody>
</table>

where \( N \) is the number of students; \( \bar{x} \) is the mean score on the final examination, \( \sigma \) is the standard deviation of the population and \( z \) is the score calculated using the formula

\[
  z = \frac{\bar{x} - \mu}{\sigma/\sqrt{N}}
\]

We formulated the null and alternative hypotheses as follows:

\( H_0: \mu = \text{or } < 57.09 \) The average score on the final exam in the experimental sections will be less than or the same as in the control sections.

\( H_1: \mu > 57.09 \) The average score on the final exam in the experimental sections will be greater than the average score in the control section.

Based on the results we can reject the null hypothesis at the 5% confidence level for the experimental sections. Our conclusion is that the interventions were successful in increasing passing rates on the final exam.

Hypothesis 2: Students in the Problem-centered Sections (Spring 2011) Will Be Better Problem Solvers

The problems on the final examination in Spring 2010 were considerably harder than those in subsequent semesters. Consequently, we could not do any analysis comparing the control and experimental sections. However, we did have data from two other Spring 2011 day sections that had the same type of word problems, and thus may serve as a comparison group. These problems were covered in all sections of the course. As can be seen in Figure 1, students in the experimental sections performed, on average, better than those in the other two sections.
When we looked at the average scores, on a scale from 0 to 5, for the experimental and comparison sections we observed that students in the experimental sections performed considerably better than those in the comparison sections on the following word problems:

**Rectangle Problem:** The length of a rectangle is four less than two times the width. The perimeter of the rectangle is 64 inches. Find the length and the width of the rectangle.

**Linear Equation Problem:** Find the slope and $y$-intercept of the line passing through the points $(2; 7)$ and $(7; –3)$. Then, write an equation for the line.

**Consecutive Integer Problem:** Find three consecutive integers such that the sum of the first two numbers is fourteen more than the third. Write and use an algebraic equation to solve this problem.

**Right Triangle Problem:** A right triangle has a hypotenuse measuring 10 in and one leg measuring 4 in. Find the length of the third side. Express your answer in simplest radical form.
Hypothesis 3: Retention of Students in Class Will Increase and a Higher Percentage of Students Will Pass the Final Exam

We examined the data in two ways. First we looked at the number of students who completed the course compared with the number of students who were enrolled at the beginning of the semester. Then we compared that number with the number of students who passed the final exam.

Table 2: Consecutive Integer Problem

<table>
<thead>
<tr>
<th></th>
<th>Control (Spring 2010)</th>
<th>Distributed Practice (Fall 2010)</th>
<th>Distributed Practice with Problem Centered Learning (Spring 2011)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number completed*</td>
<td>36/48 = 75%</td>
<td>37/52 = 71%</td>
<td>43/57 = 75%</td>
</tr>
<tr>
<td>Number enrolled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number passing final</td>
<td>18/36 = 50%</td>
<td>21/37 = 57%</td>
<td>26/43 = 60%</td>
</tr>
<tr>
<td>Number completed</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Number completed refers to any student who completed the course and received an academic grade A–F, R

Although there is essentially no difference in the retention rate across the three sections, there is a steady increase in the percentage of students who passed the final exam. This could account for the significant difference found between the average scores on the final exam between the experimental and control sections.

Conclusions

In summary, this study found evidence to support all three of the hypotheses. It appears that both experimental treatments, distributed practice and problem-centered learning, had a significant impact on learning and retention of concepts and skills as evidenced in the final examination results.
Problem-centered learning contributed to students’ increased ability to solve word problems. Finally, students in the experimental section had higher attendance rates, as demonstrated by their higher rate of course completion.

When considering these results, there are several limitations that must be considered. For example, the analyses for hypotheses two and three employ descriptive statistics rather than tests of statistical significance. These analyses could have also benefited from the inclusion of control variables to account for student background characteristics that may have differed across class sections and semesters. Finally, there is the possibility that differences across cohorts, for example, in the content of the final exam or in instructors, may have biased results. Nevertheless, this study provides preliminary evidence that distributed practice and problem-centered learning hold promise for students in remedial math courses at CUNY.

Next Steps
These results will be presented at the meeting of the department. The hope is that the faculty will adopt the syllabus designed for distributed practice and will use the problems developed for the problem-centered treatment.

Professor Bates will teach one section of Elementary Algebra in Fall 2011 using distributed practice and problem-centered learning. She will also try to obtain final examination results and withdrawal rates for each of the day sections. She plans to follow the students in the control and experimental sections to determine whether distributed practice contributes to long term retention of concepts and skills. She will try to determine whether there are statistical tests of significance for inverted bell curve distributions. (Professor Forman retired from BCC so will not participate in follow-up activities.)

In Spring 2012, she will analyze the results and write a paper for publication in an appropriate journal such as the MathAMATYC Educator. Additional funding to enlarge the project will be sought from various agencies such as NSF’s TUES program or from the US Department of Education’s Hispanic Serving Institutions—Science, Technology, Engineering, or Mathematics program.

In the Fall of 2012, Dr. Bates will follow up with students who were in the distributed practice group to gather data on their retention of the course material. This will allow for a test of the hypothesis that distributed practice increases long-term retention.

Acknowledgements
A special thank you to Dr. Milena Cuéllar who assisted us in the statistical analysis.

References
COMAP: Mathematics Instructional Resources for Innovative Educators (http://www.comap.com)
Improving Undergraduate Mathematics Learning Grant:  
The Effect of Small-Group Homework Tutoring on Remedial Mathematics Learning

Assistant Professor Alice Welt Cunningham, Mathematics Department  
Assistant Professor Olen Diaz, Mathematics Department  
Hostos Community College
This report summarizes the experiment, its results, and its dissemination, for the Improving Undergraduate Mathematics Learning grant awarded November 30, 2009, by The City University of New York Central Office of Academic Affairs, for a study of the effect of small-group homework tutoring on remedial mathematics learning.

**Background**

The research funded by the grant involved an assessment of various tutoring methods for the math lab portions of the two remedial mathematics courses, Basic Math Skills (Math 010) and Elementary Algebra (Math 020) designed by Hostos Community College to foster passing the CUNY COMPASS exit-test necessary to college-level work and graduation. Traditionally, these courses’ math labs (which constitute one of the four weekly classes for each course) have been conducted in two different formats: (1) using departmentally-prepared pencil-and-paper exercises designed to reinforce the current week’s lessons; or (2) using MathXL, Pearson Publishing’s interactive online homework vehicle derived from the respective courses’ textbooks.

**Method**

Previous research at Hostos demonstrated the effectiveness of using MathXL in increasing remedial students’ mathematics performance (Menil & Dias, 2008). In the belief that mathematics is not a spectator sport, we hypothesized that increasing the number of math lab tutors would further support student homework completion, thus increasing their mathematics performance, as measured by COMPASS results and final class grades. To this end, we studied 18 sections, nine for each of the two courses. Three sections for each course constituted the experimental cohort (E) with multiple tutors using MathXL; three sections for each course constituted the first control group (C1), with the traditional single tutor using MathXL; and three sections for each course constituted the second control group (C2), with the traditional single tutor using pencil-and-paper exercises. The experiment involved 529 students and 11 instructors. Of these 11 instructors, seven were full-time and four were part-time faculty members; all but one had previous experience teaching the relevant courses.

**Findings**

Because of state budget delays, tutors for the experimental cohorts became available only by the 6th week of the semester (or close to ½ of the way through the 14-week semester) and were hired primarily from among Hostos students with no previous teaching experience. For the same reason, tutor training in the use of MathXL, originally scheduled for before the start of classes, occurred only half-way through the semester and took place only once instead of twice. By contrast, tutors for the two control groups had begun teaching by the third week of the semester and were drawn from Hostos’ traditional tutor corps, all with experience in teaching the two remedial courses and in using MathXL.

Nevertheless, our findings, while not as robust as we might have liked in terms of the single/multiple-tutor dichotomy that was the premise of the experiment, strongly support the use of online homework-completion tutoring as a means of increasing student mathematics performance in these courses. While the C1 control group (using a single tutor and MathXL) outperformed the experimental group (using a multiple tutor and MathXL) in each course on all three COMPASS criteria (certification rate, certified pass rate, and whole-class pass rate), the experimental group, notwithstanding its late tutor start date, significantly outperformed the second control group (using a single tutor and pencil-and-paper exercises) in the Basic Math Skills course at a 005 significance level. (See Table 1.) More importantly, although both the C1 MathXL control group and the C2 pencil-and-paper control group
began their tutoring classes at the same point in the semester, the online homework group significantly outperformed the pencil-and-paper control group on all three COMPASS criteria. (See Table 2.) These results both corroborate previous research at Hostos regarding the impact of interactive online homework in improving student performance (Menil & Dias, 2008) and support the importance of homework-completion tutoring, the earlier begun, the better.

Additional results, including a study of final grades, gender, and ethnicity, were inconclusive. However, in each course, math lab attendance for the two online homework-completion groups (E and C1) far exceeded that of the pencil-and-paper group. (See Figures 1-2, Math 010; and Figures 3-4, Math 020). Moreover, as measured by final course grade, mathematics performance for the combined experimental and control online homework-tutored cohorts in each of these courses showed a positive linear relationship with math lab attendance at a 0.01 significance level. (see Tables 3 and 4.) These attendance data extend the previous research noted above and further support the efficacy of interactive online homework in fostering problem-solving rather than passive note-taking in remedial mathematics courses. Math is not a spectator sport!

**Related Projects and External Funding**

Because the state budget delay precluded effectuation of our research in accordance with its design, we elected not to continue the single/multipletutor research beyond the single-semester experiment for which the grant was awarded. For the same reason, we did not seek additional funding for this particular project. However, because of the success of the two online homework completion math lab cohorts (E and C1), additional multi-section research, funded by Hostos and led by a senior Hostos mathematics professor, is being conducted during the Fall 2011 semester in the Basic Math Skills course. Based on our results, further research on the effects of using online homework to achieve active student problem-solving rather than passive note-taking (e.g., Hinds, 2009) is surely warranted.

**Dissemination**

Our findings have already been widely disseminated through presentations and publications, with further work in progress.

**Presentations**

Our preliminary findings, regarding the COMPASS results described in Table 1, were disseminated to the CUNY mathematics research community at a CUNY-wide symposium held for this purpose at the Graduate Center this past February 18, 2011. We presented our preliminary findings to the Hostos Mathematics Department at a meeting held this past May 12, 2011, and to the entire Hostos teaching community at a Professional Development Initiative seminar held this past May 31, 2011. Our final results are scheduled for presentation the Hostos Mathematics Department on October 25, 2011, and at the Tenth Annual CUNY Information Technology Conference at John Jay College of Criminal Justice on December 2, 2011.

**Publications**

Initial Results. Our preliminary findings regarding the COMPASS results were summarized in an article (attached). This article was published in the Spring 2011 edition of Touchstone, Hostos’ annual year-end journal describing faculty research during the preceding academic year, as well as in the year-end Mathematics Journal, the vehicle of the Hostos Mathematics Department that serves the
same function. In addition, copies of the article were sent at their request to the two researchers at Columbia University’s Community College Research Center who observed my Elementary Algebra class on February 18 of this past year, as well as to my contacts at Pearson Publishing, again at Pearson’s request.

Final Results. The analysis of our additional findings has been summarized in an article, Math is Not a Spectator Sport: The Effect of Online Homework Tutoring on Community College Remedial Mathematics Performance (attached). A condensed version of this article is under consideration for publication for the forthcoming technology issue of a peer-reviewed journal. At the request of Pearson Publishing, a copy of the condensed article was made available for Pearson’s on-campus training of faculty and staff on August 30 and 31 (Sestak, 2011). A copy of the final results was sent to the Community College Research Center as well.

Summary

The net effect of our research under the Improving Mathematics Learning grant, while not as robust as we would have liked in terms of the multiple/single tutor dichotomy, has succeeded well beyond our expectations under the circumstances. Despite the state budget complications that precluded the effectuation of the research in accordance with its design, we were able to corroborate earlier research regarding the effectiveness for community college remedial mathematics students’ performance of interactive textbook-based online homework (Menil & Dias, 2008). Our own research extended these findings to student attendance at tutoring classes as well as to substantive performance, thus further substantiating the effectiveness of active problem-solving over passive note-taking in improving both student engagement and performance.

The tables and figures immediately following set forth the substance of our findings.

References


### Table 1: Performance comparison of experimental and pencil-and-paper lab groups (using a right-tailed z-test for the respective group percentages)

<table>
<thead>
<tr>
<th>Groups</th>
<th>Certification rate</th>
<th>Certified COMPASS pass rate</th>
<th>Class COMPASS pass rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental (n=89)</td>
<td>51%</td>
<td>67%</td>
<td>35%</td>
</tr>
<tr>
<td>Control-2 (n=86)</td>
<td>45%</td>
<td>52%</td>
<td>24%</td>
</tr>
<tr>
<td>Percent point difference</td>
<td>6 percentage points</td>
<td>15 percentage points</td>
<td>11 percentage points</td>
</tr>
</tbody>
</table>

\[
\% \text{ Diff} = \frac{x_1^2 - x_2^2}{(x_1^2 + x_2^2)/2} \times 100
\]

<table>
<thead>
<tr>
<th>P-value</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.201</td>
<td>N_1 = 89 X_1 = 46</td>
</tr>
<tr>
<td></td>
<td>N_2 = 86 X_2 = 39</td>
</tr>
<tr>
<td>0.028</td>
<td>Significant at 0.05 level</td>
</tr>
<tr>
<td>0.0484</td>
<td>N_1 = 89 X_1 = 32</td>
</tr>
<tr>
<td></td>
<td>N_2 = 86 X_2 = 21</td>
</tr>
</tbody>
</table>

### Table 2: Performance comparison of MathXL and pencil-and-paper control groups (using a right-tailed z-test for the respective group percentages)

<table>
<thead>
<tr>
<th>Groups</th>
<th>Certification rate</th>
<th>Certified COMPASS pass rate</th>
<th>Class COMPASS pass rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control-1 (n=90)</td>
<td>56%</td>
<td>81%</td>
<td>43%</td>
</tr>
<tr>
<td>Control-2 (n=86)</td>
<td>45%</td>
<td>52%</td>
<td>24%</td>
</tr>
<tr>
<td>Percentage point difference</td>
<td>11 percentage points</td>
<td>29 percentage points</td>
<td>19 percentage points</td>
</tr>
</tbody>
</table>

\[
\% \text{ Diff} = \frac{x_1^2 - x_2^2}{(x_1^2 + x_2^2)/2} \times 100
\]

<table>
<thead>
<tr>
<th>P-value</th>
<th>Significance</th>
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<tbody>
<tr>
<td>0.066</td>
<td>N_1 = 90 X_1 = 51</td>
</tr>
<tr>
<td></td>
<td>N_2 = 86 X_2 = 39</td>
</tr>
<tr>
<td>0.0000244</td>
<td>Significant at 0.10 level</td>
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<tr>
<td></td>
<td>N_1 = 90 X_1 = 73</td>
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<td></td>
<td>N_2 = 86 X_2 = 45</td>
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<tr>
<td>0.004</td>
<td>N_1 = 90 X_1 = 39</td>
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<tr>
<td></td>
<td>N_2 = 86 X_2 = 21</td>
</tr>
<tr>
<td></td>
<td>Significant at 0.01 level</td>
</tr>
</tbody>
</table>
**Figure 1:** Basic Math Skills Math Lab attendance by cohort

- **Blue** Experimental
- **Orange** Control-1
- **Purple** Control-2

**Figure 2:** Basic Math Skills Math Lab attendance by online and pencil-and-paper cohorts

- **Blue** Experimental + Control-1
- **Orange** Control-2

**Table 3:** Basic Math Skills grade/lab attendance correlation ($r$: linear correlation coefficient, using linear regression t-test)

<table>
<thead>
<tr>
<th>Grades</th>
<th>A</th>
<th>B</th>
<th>B-</th>
<th>R</th>
<th>F</th>
<th>P-value</th>
<th>r</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>E + C1</td>
<td>11</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>5</td>
<td>0.0064</td>
<td>0.969</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>Grades</td>
<td>A</td>
<td>B</td>
<td>B-</td>
<td>R</td>
<td>F</td>
<td>P-value</td>
<td>r</td>
<td>p</td>
</tr>
<tr>
<td>C2</td>
<td>10</td>
<td>6</td>
<td>8</td>
<td>4</td>
<td>1</td>
<td>0.0036</td>
<td>0.9025</td>
<td>&lt; 0.05</td>
</tr>
</tbody>
</table>
**Figure 3:** Elementary Algebra Math Lab attendance by cohort

- **Experiment**
- **Control-1**
- **Control-2

**Table 4:** Elementary Algebra grade/lab attendance correlation (r: linear correlation coefficient, using linear regression t-test)

<table>
<thead>
<tr>
<th>Grades</th>
<th>A</th>
<th>B</th>
<th>B-</th>
<th>R</th>
<th>F</th>
<th>P-value</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>E + C1</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>7</td>
<td>5</td>
<td>0.003</td>
<td>0.981</td>
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<tr>
<td>Grades</td>
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<td>R</td>
<td>F</td>
<td>P-value</td>
<td>r</td>
</tr>
<tr>
<td>C2</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0.16</td>
<td>0.73</td>
</tr>
</tbody>
</table>

**Figure 4:** Elementary Algebra Math Lab attendance by online and pencil-and-paper cohorts

- **Experiment + Control-1**
- **Control-2**
An Internet-based System of Homework for Precalculus

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Professor Stanley Ocken, Department of Mathematics
Professor Irvin Schonfeld, Department of Psychology
The City College of New York
Objective

The goal of the proposal was to test whether providing online homework using the Maple TA system would significantly improve outcomes in the precalculus course (Math 19500). The primary measure of success was the score on a uniformly administered and graded final exam.

Design

All day-time sections of Math 195 in fall 2010 and spring 2011 were eligible to participate in the study. Half were chosen for the experimental group using Maple TA and half were designated as controls in which all homework was done using traditional paper-and-pencil methods. In total, twelve (seven fall plus five spring) sections were randomly assigned to each protocol. Instructors who taught two sections had one section randomly assigned to the control group and the other to the experimental groups. PIs Grossman and Ocken each taught one experimental section. Otherwise, the control and experimental sections were assigned at random.

Preliminary Work

The experiment required the development of several important components. The most important was the data base of online homework. Experimental and control sections were to do substantially the same homework, which was drawn from the text, Precalculus: Mathematics for Calculus, 5th ed, by Stewart, Redlin and Watson, and Algebra Notes prepared by Prof. Ocken. The PIs, with the assistance of two summer associates, developed a data base of more than 550 problems, duplicating in randomized form essentially all problems assigned from the text. These included a number of innovative Flash-based graphing tools developed by Prof. Ocken for implementing and evaluating student graphical responses, when this was the principal goal of an exercise.

In addition, in order to provide a uniform baseline measure of student skill level, during the spring and summer of 2010 we tested and modified an algebra diagnostic test that was given at the beginning and end of the course in all sections.

Data

As described in the proposal, in addition to scores and other data generated by the experiment, we obtained background data on each student in order for the statistical analysis to control for significant covariates. The data was obtained from the CCNY Office of Institutional Assessment and directly from the Student Information Management Systems (SIMS). Variables included ethnicity, financial aid status, admit status, HS average, CCNY GPA, grades on NYS Math Regents Exams, Math and Verbal SAT scores, COMPASS algebra, college algebra, and trigonometry scores, and last math course taken at CCNY and the corresponding grade.

Basic Statistical Comparisons of Experimental and Control Sections

On assorted statistical measures such as ethnic composition, economic status as defined by financial aid, initial score on the diagnostic test, HS average, Math SAT, and Compass Algebra score, there were no significant differences between the experimental and control groups.

On the main dependent variable in the study, the final exam score, the statistical analysis provided by Dr. Schonfeld for each semester showed that there was no statistical difference in performance on the final exam between the two groups, when we controlled for the two most significant predictors: the diagnostic test score and the Compass College Algebra score.
For students using Maple TA, where we had accurate records of homework completion, homework success predicted performance on the final exam, controlling for covariates reflecting precourse knowledge of mathematics. Moreover, progress over the fall 2010 implementation was evident in substantially higher rates of homework completion in Maple TA classes in the spring semester.

Further Plans

While this implementation did not achieve the goals envisioned by the PIs, we think that a more robust homework system may yet yield significant improvements in outcomes. To that end, Prof. Ocken and Dr. Tamara Kucherenko of the CCNY Mathematics Department, under the auspices of a Title V grant administered by the College, will oversee a continuation of this study using the Enhanced Web Assign platform developed by the textbook publisher, Cengage. This system provides substantially more support for students in the way of online tutorials and detailed problem solutions. It also allows students to focus on problems they have found difficult, without having to expend effort on material they have already mastered.

Of course, it is possible that whether homework is done online or in the traditional manner makes no difference. Such a conclusion would be premature at this stage, without thoroughly evaluating another system that corrects some of the deficiencies we identified in Maple TA.

Statistical Study

A full statistical report can be obtained from Prof. Grossman or Prof. Schonfeld. What follows is an excerpted version of that report.

Student Performance in Math 19500 during the Spring 2011 Semester at the City College of The City University of New York
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Background & Findings

A total of 356 City College students were enrolled in ten sections of Math 19500 during the spring 2011 semester. Five sections were exposed to Maple TA for all homework and five, i.e., the control sections, completed their (essentially similar) homework using paper-and-pencil methods traditionally employed in Math 19500. The ten sections were taught by nine different instructors, one of whom taught both a Maple TA class and a control class.

The experimenters hypothesized that students enrolled in the Maple TA sections would earn higher grades on the common final exam, as well as higher course grades. I put greater emphasis on the hypothesis as it bears on final exam grades because the final examinations were graded with the help of a scoring rubric, and each professor was limited to grading two or three exam questions. To evaluate the hypothesis, I conducted a number of statistical tests to compare the performance of the two groups of students.

The chi-square test indicated that the distributions of course grades in the two sets of classes did not significantly differ, $\chi^2(5) = 6.10$. 
At the end of the semester, students took a common final examination. Among the Maple TA classes, the mean grades on the final exam were below 70. In one control class, the mean grade was above 70; in three other control classes, the means were below 60. I conducted a $t$-test for independent samples comparing one Maple TA class to a control class when the pair of classes was taught by the same instructor. Conducting an overall $t$-test in which all Maple TA and all control students are compared is statistically unwarranted because of the nested nature of the data (I take up the matter of nesting below). The mean difference in final exam performance in the two classes taught by the same instructor was not significant.

Students enrolled in Math 19500, like students at any college that runs multiple sections of the same course, are nested within sections of the course. Nesting often leads to a condition in which individuals in any one group are likely to be more similar to each other than they are to individuals in another group. Hierarchical linear modeling (HLM) procedures take into account the nested nature of much of the data investigators collect, and the intra-group similarities in those data. Traditional approaches to data analysis, for example, multiple linear regression (MLR), ignore the nested nature of data and, as a consequence, yield less accurate parameter estimates and estimates of standard errors (Raudenbush & Bryk, 2001; Schonfeld & Rindskopf, 2007).

The intraclass correlation (ICC) for final exam scores among Math 19500 students was 0.08 (and 0.06 for course grades), indicating that there was sufficient within-group similarity that it was necessary to avoid using MLR and instead turn to HLM (when the ICC is near zero, MLR and HLM yield similar results).

In the analyses that follow, I evaluated the differences between the Maple TA and control classes using HLM. I also adjusted for a number of precourse covariates reflecting student math knowledge. These covariates potentially account for differences between the students in the Maple TA and control classes. The covariates included SAT math, the various CUNY Compass exams, and a diagnostic exam developed by Profs. Grossman and Ocken. A separate study of the diagnostic examination indicated that the exam had satisfactory psychometric properties (e.g., $\alpha = .81$, positive correlations with other measures of mathematics knowledge). In the HLM analyses, the type of course taken was represented by a dummy variable in which enrollment in a Maple TA class was coded as 1 and enrollment in a control class, 0.

In HLM analyses shown in Table 1, final exam scores were regressed on the 0-1 dummy variable representing attendance in a Maple TA or control class (model 1). The table shows that although students in Maple TA classes scored, on average, 2.7 points higher than their peers in the control classes, the difference was not statistically significant. The remaining analyses involved adding and trimming covariates until I identified the “best” set of covariates in terms of explaining performance on the final exam. The best explanatory covariates were the diagnostic test, the Compass College Algebra test, and the Compass Trigonometry exam. SAT math was at best marginally ($p < .14$) related to performance on the final exam when the diagnostic test was controlled but its coefficient sank to close-to-zero when the Compass exams were controlled. In any event, no adjustment for covariates would reveal a significant effect for Maple TA.
One final set of analyses examined homework performance in the Maple TA classes. On average, Maple TA students completed 60% of their assigned homework, up from 40% in the fall 2010. Fifty-six percent of Maple TA students completed at least 60% of their homework assignments, up from 26% in the fall 2010. Number of homework problems correctly solved correlated moderately with final exam performance, \( r = 0.53 \), which is very close to the median within-group correlation \( (r_{mdn} = 0.56) \). To adjust for clustering, I regressed exam performance on number of homework problems solved using HLM. Homework performance was highly significant (\( \beta = 0.068, se = 0.009, p < .001 \)). I also determined if the relation of homework performance to final exam performance would withstand controls for the chief covariates reflecting precourse math achievement, Compass College Algebra and Trigonometry scores and the scores on Profs. Grossman and Ocken’s diagnostic examination. The impact of homework success remained significant and largely unchanged (\( \beta = 0.062, se = 0.011, p < .001 \)).

### Comments

The hypothesis was not supported. There was no significant mean difference between the groups on final exam grades. Nor was there a significant mean difference between the groups in course grades. Because of its more objective nature, findings bearing on the final exam were deemed more important than the findings bearing on course grades.
Maple TA had a built-in stumbling block. Although students could redo assignments to improve their grades, the software provided students with a less-than-optimal opportunity to do so. A student had to redo an entire assignment, not just the items he or she got wrong, in order to get credit toward the homework grade. If a student wished to redo only the items he or she initially got wrong, this could be done only anonymously, with no credit. This problem with Maple TA likely undermined student motivation.

Despite the abovementioned stumbling block, performance on the homework was related to performance on the final exam, controlling for precourse mathematics knowledge. Because parallel data on homework performance in the control classes was not collected, I could not compare the relation of homework performance to final exam performance in the two groups.

Dissemination. It is premature to disseminate the findings. I stress again that the development of the interactive homework software is an iterative process. The faculty need more time to try out the next iteration of the interactive homework package expected to be implemented in the fall 2011. As we want to show patience with students having weak backgrounds in mathematics, we need to show patience with developers in shaping the intervention to help their targeted students.

References


Accelerated WARM UPS
Workshop Approach to Remedial Mathematics Using Problem-Solving

Assistant Professor G. Michael Guy, Department of Mathematics and Computer Science
Assistant Professor Jonathan Cornick, Department of Mathematics and Computer Science
Associate Professor Robert Holt, Department of Mathematics and Computer Science
Instructor Andrew Russell, Department of Mathematics and Computer Science
Queensborough Community College
National Significance of Remediation

In 2010–2011, approximately 3.2 million students will graduate high school (Snyder & Dillow, 2011). Unfortunately, many will graduate underprepared for the rigors of college. While future improvement in high school preparation is a cherished goal, the current level of student readiness requires a collegiate solution, particularly at open-enrollment community colleges. In fall 2005, of the 1.3 million students enrolled in math courses at two-year colleges, 57% were enrolled in a developmental course (Blair, 2006). In theory, the developmental math sequence is designed to prepare students to succeed in college-level math. In practice, it often serves as a trap from which most students never escape, and those who do often do not persist to complete a credit-bearing gatekeeper course. An analysis of 57 community colleges by Bailey, et al. (2010) found that only 33% of students referred to developmental math completed the sequence within 3 years and only 20% passed the gatekeeper course. Their analysis revealed a key barrier to completing the sequence is the persistence of students to enroll in the next course. Students who successfully complete a developmental math course have higher fall to spring retention than those who enroll but do not successfully complete it (Fike & Fike, 2008). Developmental education costs an estimated $1.9-$2.3 billion annually at community colleges (SAS, 2008). Students also suffer additional cost using financial aid. The ongoing discussion about remediation occurs while the current knowledge base is insufficient to inform the decisions of policy makers, educators, scholars, and students (Levin & Calcagno, 2008).

Remediation at CUNY

With a large urban enrollment, CUNY is a microcosm of national trends. Recently, the Community College Research Center conducted an analysis of CUNY’s community colleges. Between fall 2004—spring 2008, 74,772 first-time students entered CUNY community colleges. These students were followed until fall 2009. During this period, 82% of students were referred to some type of developmental education including 64% in math. Of the students who enrolled, persistence to completing the gatekeeper course varied significantly based on the severity of remediation need. The most frequent starting point (49%) enrolled at two levels below credit bearing, and only 13% of them persisted and passed the gatekeeper course. At CUNY, students who fail a remedial math course in their first semester are more than four times as likely to drop out as those who pass the course (Jaggars & Hodara, 2011).

Intervention at Queensborough Community College

Students entering Queensborough Community College (QCC) are placed into mathematics courses based on their scores on the COMPASS mathematics exam: If they score less than 30 on M1 (arithmetic), they are generally required to take MA-005, a semester-long remedial arithmetic course. During the Fall 2008-Spring 2009 academic year, 1,872 students enrolled in our lowest-level remedial arithmetic course. Unfortunately, only about 37% (692) of those students successfully completed it. Anecdotal evidence suggested that many students who “almost” passed M1, failed or dropped out of MA-005 because they were discouraged by a course which attempts to again teach them material they have already seen for several years in high school. These students often do not reenroll in the following semester and fail to make progress towards a degree despite their initial desire to do so. To meet the needs of such students and prepare them for a credit-bearing course, a new course, MA-005M (Arithmetic WARM UPS), was developed in Summer 2009 and offered for the first time in the Fall 2009 semester. Development of this course was supported first by the QCC Office of Academic Affairs and then by an Improving Math Learning Grant from CUNY.
The Arithmetic WARM UPS (Workshop Approach to Remedial Mathematics) model is a 4-week, 20-hour workshop which includes 4 computer lab hours. In this model, the emphasis is on students engaging with problem solving in order to improve their arithmetic skills. To enable the radical change in classroom practices, a new textbook, Arithmetic WARM UPS (Cornick, Guy, Holt, & Russell, 2010) was written. The book tightly aligns with the new structure, curriculum and pedagogical innovations.

On each of the first four meetings, students complete one Skill Sheet that focuses on only a few topics. During these meetings, an instructor presents quick refreshers at the board, and then turns the focus to the students to engage in problem solving. Indeed, most of the period is spent with the student completing problems and not the instructor lecturing. A mantra often repeated throughout this project is, “The instructor must sometimes stop teaching in order for the student to start learning.” After the introductory refreshers, the instructor then becomes an active participant circulating through the classroom and engaging students who are struggling. At this point the instructor offers differentiated instruction to address the learning needs of one or a small group of students. Students have many resources at their disposal: they can remind themselves of what to do next by consulting the Help Pages in the book, ask their instructor for assistance, discuss with a fellow classmate or seek out help from a friend or tutor. After the first four meetings, one Mixed Worksheet is completed per meeting with little to no lecturing to the entire class. The Mixed Worksheets contain problems from all curricular topics, and they are not arranged or subdivided in any particular order. In order for students to begin answering these questions, they must first identify which of their skills they will need to solve the problem.

In order to earn a passing grade, students are required to earn a score of 30 on the COMPASS Arithmetic (M1) test. After completing 20 hours of class time, students are given their first opportunity to take the exam. If a student scores a passing grade, a 30 or higher, they complete their arithmetic requirements and are allowed to take algebra the following semester. If they do not pass, then they continue for 20 more hours of instruction by a peer tutor in the Math Learning Center. In addition to a changed classroom format, students are afforded multiple opportunities to earn a passing grade during a single semester.

Since the course requires only 20 hours of class time, students start the class at three times throughout the semester. Module A begins the course immediately, module B begins the course approximately 5 weeks into the semester, and module C begins approximately 10 weeks into the semester. There is an additional post semester workshop offered in the Math Learning Center for students to have an additional opportunity to pass before the next semester began. Each module has a maximum enrollment of 20 students and one instructor is thus able to teach approximately 60 students per semester.

**Brief References to Relevant Existing Literature**

It seems intuitive that students with more deficiencies should require more semesters of remediation. This seemingly intuitive notion has guided the development of many, multi-level remedial sequences. However, Edgecombe (2011) presents myriad arguments against an elongated remedial sequence. Edgecombe argues that many students fail to complete the remedial sequence and a credit-bearing course simply because they drop out of the sequence at every available exit point. She argues that the acceleration of the sequence by giving students an opportunity to earn a passing grade in a gatekeeper course sooner will result in higher rates of credit completion. Additional research also suggests that the faster students progress toward a credential, the more likely they are to complete college (Bowen, Chingos, & McPherson, 2009).
Edgecombe presents several strategies for effectively accelerating the sequence. She cites numerous examples where course acceleration results in improvement of student learning outcomes. She notes that this is not always accompanied by a reduction in overall class time (2011).

Effective teaching in a developmental classroom is a challenging undertaking extending beyond purely cognitive needs (Smittle, 2003). Our practices incorporate a massive departure from teacher-centered lecture and testing and implements student-centered classroom practices that encourage high student engagement. We incorporated practices of highly structured collaborative problem solving and additional learning centered practices described by Hodara (2011). Additional existing research supporting various elements of our intervention can be found in Rutschow and Schneider (2011).

**Experimental Design**

Beginning in Fall 2009, arithmetic students with an arithmetic COMPASS M1 score of 25-29 were eligible to enroll in either the traditional arithmetic course or our newly redesigned Arithmetic WARM UPS course. Students self-selected into one of the courses. While initially enrollment in the experimental course was limited to only students in the 25-29 range, students outside this range were allowed to enroll in the course to fill remaining seats in the days immediately prior to the course start date. Factors influencing student decisions were not studied.

Instructors for each class were assigned by the Deputy Chairperson who makes all instructor assignments. There were 12 different instructors for the experimental class and 63 different instructors for the traditional course. Four instructors taught both courses during this time. The majority of instructors in both groups were adjunct instructors, but there were also full time faculty in both groups. Two authors of this report, Guy and Russell, served as experimental instructors for 3 of the 4 semesters in this study and Cornick served as an experimental instructor for approximately 20 students during the study.

We analyze the success of the new program by means of quasi-experimental methods. In this report, we present several results obtained by use of multiple logistic regression techniques. In a logistic regression technique, the statistical differences between a dichotomous dependent variable are calculated while taking into account the contributions one or more independent variables. In our results below, the dependent variable will be an indicator variable for whether students achieved some milestone success. For example, we will indicate with a 1 a student who passed the course and a 0 for a student who did not pass the course.

**Population Studied**

During the four semesters Fall 2009, Spring 2010, Fall 2010 and Spring 2011, 3,783 students enrolled in one of the two courses. Unfortunately, student outcome data is not yet fully available for Spring 2011 students and so these students are all excluded this this analysis. In order to attempt to control for preexisting differences in student populations, we limit the analysis presented here to only students who meet all of the following conditions:

1. Student has a COMPASS Arithmetic (M1) score on file. In addition, this score had to be less than a 30.
2. Student has a COMPASS Algebra (M2) score on file.
3. Student’s first math course was either traditional arithmetic (MA 005) or the experimental course (MA 005M).
4. Student’s first attempt at a math course was during the Fall 2009, Spring 2010 or Fall 2010.
These conditions limit our study to 2,306 students. For data calculation, we assigned students to the experimental group (MA 005M) or the control group (MA 005) based on the course they self-selected into on their attempt.

Only brief demographic data is presented here, and in this report, demographic data is not factored into our calculations. Table 1 contains the reported sex of each student and Table 3 contains the age of each student when entering the study.

Since the new class was still unproven, we initially opened it to only students whose COMPASS M1 scores were in the 25–29 range. This did create a disparity in this measure of student test characteristics, as seen in Table 4. In our analysis below, we make use of regression techniques with hope of accounting for some of those differences. Roughly speaking, in a regression analysis students with “similar” characteristics are compared based on each independent variable. Thus the outcome accounts for some of the differences between the two groups by factoring these differences into the final results. See H Plans for Future Research for a discussion of future plans to analyze the data.

**Measures of Success**

We now explore various measures of success of our students. In each of the regression calculations below, the student’s Arithmetic (M1) and Algebra (M2) COMPASS scores are included as independent variables in the logistic regressions. In addition, an indicator variable, course1_en, is assigned a value of 0 if the student first enrolled in the traditional arithmetic course (MA 005) and value of 1 if the student first enrolled in the experimental arithmetic course.

Note: The authors of this report are not statisticians, despite being mathematicians. The attempts at analysis here using advanced statistical methods may well have gone astray, and comments about the appropriateness and execution of these tests are welcomed. More simplistic cross tabulations are also included with each analysis.

**Passing Arithmetic on First Enrollment**

For this measure of success, we define success as earning a score of 30 or higher without enrolling in arithmetic again. This indicates that the student earned a 30 or higher either during the regular semester or during a post semester workshop in the Math Learning Center. A simple cross tabulation Table 6 shows that the pass rate was approximately 44% for the control group versus approximately 73% for the experimental group. Fisher's exact test indicates that this is a statistically significant difference.

The output from the logistical regression is included in Table 5. This output indicates a logit of .5467034 for students enrolled in the experimental group. The p-values for all coefficients and the model indicate statistical significance at the 0.001 level.

We pause momentarily to explain an interpretation of the logit values from Stata as summarized on UCLA's Academic Technology Services Website. A logit value of .546703 is the log of the odds ratio between the experimental group and control group when taking into account differences between the entering COMPASS scores of the students. We can translate this into odds by taking the exponential. Thus the odds are exp(.546703)=1.727548. This can be interpreted to say that the odds of passing arithmetic in the experimental group are roughly 73% higher than those in the control group. Again, the regression techniques take into account the differences in the population.
SAME SEMESTER MATH RETENTION

Many studies have indicated that the largest barrier to student success is the student actually staying enrolled and not quitting. In this course, we hoped that the shorter, yet more intensive, requirement for the student to complete the class would result in an increase in retention.

Students who enroll in the traditional arithmetic course are required to complete approximately 14 weeks before an opportunity to take the COMPASS exam. Certainly a student who is still enrolled at the 14th week and takes the exam should be considered retained. As a result, we assign students in the control group a value of 1 (success) if they take the COMPASS exam after enrolling in the course.

For students in the experimental group, retention is less obvious. Recall that students begin this course at 3 distinct times throughout the semester. Students who begin during module A have their first opportunity to take the COMPASS after approximately 4 weeks of class. Those who pass have completed the course. Certainly those students should be assigned a value of 1, and we do so. However, students from module A who do not pass are afforded an opportunity to continue with the class and take the exam again 4 weeks later. The dilemma is whether we should require a student to take the exam a second time (or third and even forth available opportunities) to be considered retained or is taking the exam at least once sufficient? To further complicate matters, students who enroll in module B must wait 5 weeks before even beginning class, then take class for 4 weeks before being allowed to take the exam. Those who pass are certainly counted as retained. But those who didn’t pass now fall into the same situation as those in module A who did not pass, but are now allowed another opportunity after taking 4 weeks of additional class. Module C has similar complications.

For simplicity, in this report, we offer the following definition. We define a student in the experimental group to be retained if they take the COMPASS exam at least once following enrollment in the course. This indicates they took the exam either approximately 4 weeks, 9 weeks or 14 weeks during the semester, or in the post semester workshop that followed. In a future analysis, we will examine other possible definitions of retention.

The cross tabulation Table 8 shows that retention under this definition is approximately 65% for the control group and approximately 84% for the experimental group. Fisher’s exact test indicates that this is a statistically significant difference.

The results of the logistical regression are in Table 7. The logit for students in the control group is .7794594. As before, we can take the exp (.7794594) = 2.180293 and interpret this to mean that the odds of retention are approximately 118% greater for the experimental group.

ENROLLMENT IN FOLLOWING MATH COURSE

Unfortunately, many students who pass one course in a sequence never register for the following course. A detailed discussion of this phenomenon is available in many sources including Edgecombe (2011), Bailey et al. (2010), Jaggars and Hodara (2011), and Hern (2010).

We study the enrollment behavior here. Since our data is incomplete for the Spring 2011 term, we limit our analysis of this indicator to only students enrolling in our study during Fall 2009 and Spring 2010 (n = 1,528). A cross tabulation Table 9 indicates a statistically significant difference of approximately 58% from the experimental group and only 44% from the control group enrolled in developmental algebra.
In this case, the logistical regression is not statistically significant (likely due to lower population sizes) and is not presented.

**PASSING THE FOLLOWING MATH COURSE**

One hopes that the hard work in creating a new, more successful class will result in gains beyond the single semester of the new course. But as with the previous result, we cement a pattern that is highly predictable from examining existing research (eg Weissman et al., 2011). Succinctly stated, single semester interventions result in single semester gains. The enrollment numbers alone are unfortunate, but the passing rates are even more disturbing.

We again limit ourselves to the 1,528 students who enrolled during the Fall 2009 and Spring 2010 terms. A cross tabulation Table 10 indicates a statistically insignificant comparison of approximately 15% for the control group and 19% for the experimental group.

One should note that many of these students had only a short window in which to pass arithmetic, enroll in and then pass algebra. In order to further highlight the issues at hand, we restrict ourselves to only the 881 students enrolling in Fall 2009. These students had the longest timeframe to complete the sequence of any in our study. A cross tabulation Table 11 again indicates a statistically insignificant comparison of approximately 18% for the control group and 19% for the experimental group. While the differences are statistically insignificant, it is evident that as time progresses, the gains from a single semester intervention tend to fade and those in the treatment group and control group have more similar outcomes.

**SUPPORTING ROLE BY MATH LEARNING CENTER**

Students in the experimental course who did not pass on their first attempt were eligible to attend a workshop in the Math Learning Center and take the COMPASS again after 20 additional class hours led by a peer tutor. There were 284 (of 693) experimental students did not pass the COMPASS on their first attempt. A total of 116 of these students attended a workshop and took the COMPASS a second time. Of those 116 students, 95 students (82%) earned a passing grade for the course. In sum, this indicates that while only 116 of the 284 (41%) made use of the additional supporting workshop, it was extremely successful for those who did participate.

**Plans for Future Research**

There are a number of issues which we have not addressed in this report. For example, we have not addressed any differences between experimental students enrolling in modules A, B and C. We would like to study the differences in outcomes when accounting for this difference in start time. Moreover, we have merely listed various demographic details here, but have made no attempt at determining any difference in effectiveness among these subpopulations. We have yet to analyze if students with additional remediation deficiencies in other areas are equally well served under this newer model. In addition, Spring 2011 students were not included in this analysis. We intend to collect the remaining data, and study the behavior of these students in the Fall 2011 as well. In summary, there are quite a few questions which this report has not yet addressed. We will address these issues, and others, and submit our study for peer reviewed publication(s).
Future Practice

We have already made the decision to scale up this new course. In the Fall 2011 semester, it is now the dominant mode of arithmetic offering. There are over 1,000 students enrolled in MA 005M this semester, and we will continue to study the effectiveness of this new course in the wider population.

In addition, it is evident that a single semester intervention results in a single semester gain. As a result, our attention has shifted from focusing on a single course to the entire developmental sequence. Several experiments and projects are now underway to expand on our success in this course, and cover the entire sequence. Several of these projects are supported by competitively awarded QCC and CUNY grants. Additional funding opportunities for external resources are also under consideration.

Acknowledgements

Any complete listing of all those who played a significant role in this project would go on longer than we are allotted for our entire report. However, we must thank our Department Chairperson Dr. Mona Fabricant for getting this projected started by sharing her great ideas and helping secure initial funding. We offer our highest praise to Ms. Elizabeth Nercessian, the director of the Math Learning Center, for playing such a strong supporting role in this new course. Our Deputy Chairperson Ms. Sandra Peskin and Mr. Ed Molina also contributed a wealth of experience and helped make our project a success. We also offer our appreciation to our many colleagues, including faculty, staff and administrators, who supported and enabled this work. Our students have truly benefited from your support. As our work continues, we look forward to your continued support.

All data in this report was generously provided by the CUNY Central Office of Institutional Research with the oversight of QCC’s IRB. Special thanks to Dean David Crook, Cheryl Littman and Stephen Sheets for their very thorough assistance with our data requests.

Tables & Figures

Table 1: Student Sex

<table>
<thead>
<tr>
<th>Course</th>
<th>Fall 2009</th>
<th>Spring 2010</th>
<th>Fall 2010</th>
<th>Grand Total</th>
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</thead>
<tbody>
<tr>
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<td>579</td>
<td>344</td>
<td>235</td>
<td>1,613</td>
</tr>
<tr>
<td>F</td>
<td>517</td>
<td>322</td>
<td>195</td>
<td>263</td>
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<tr>
<td>M</td>
<td>316</td>
<td>130</td>
<td>120</td>
<td>693</td>
</tr>
<tr>
<td>Experimental (MA005)</td>
<td>302</td>
<td>160</td>
<td>142</td>
<td>604</td>
</tr>
<tr>
<td>F</td>
<td>317</td>
<td>59</td>
<td>120</td>
<td>89</td>
</tr>
<tr>
<td>M</td>
<td>120</td>
<td>71</td>
<td>120</td>
<td>604</td>
</tr>
<tr>
<td>Grand Total</td>
<td>881</td>
<td>647</td>
<td>778</td>
<td>2,306</td>
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Table 2: Meaning of semester 1 variable in Statistical Output

<table>
<thead>
<tr>
<th>Semester1 Value</th>
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<tr>
<td>2</td>
<td>Fall 2010</td>
</tr>
<tr>
<td>3</td>
<td>Spring 2011</td>
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Table 3: Student Age

<table>
<thead>
<tr>
<th>Course</th>
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<th>Spring 2010</th>
<th>Fall 2010</th>
<th>Grand Total</th>
</tr>
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<td>Control (MA005)</td>
<td>579</td>
<td>517</td>
<td>517</td>
<td>1,613</td>
</tr>
<tr>
<td>17–21</td>
<td>469</td>
<td>336</td>
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<td>22–26</td>
<td>63</td>
<td>117</td>
<td>62</td>
<td>242</td>
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<td>27–31</td>
<td>26</td>
<td>25</td>
<td>21</td>
<td>72</td>
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<tr>
<td>32–36</td>
<td>9</td>
<td>17</td>
<td>3</td>
<td>29</td>
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<tr>
<td>37–41</td>
<td>5</td>
<td>10</td>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>42–46</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>9</td>
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<td>47–51</td>
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<td>4</td>
<td>4</td>
<td>13</td>
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<td>52–57</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
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<td>Experimental (MA005)</td>
<td>302</td>
<td>130</td>
<td>261</td>
<td>693</td>
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<td>17–21</td>
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<td>245</td>
<td>578</td>
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<td>22–26</td>
<td>38</td>
<td>26</td>
<td>8</td>
<td>72</td>
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<td>2</td>
</tr>
<tr>
<td>Grand Total</td>
<td>881</td>
<td>647</td>
<td>778</td>
<td>2,306</td>
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</table>

Table 4: Initial COMPASS Test Scores

<table>
<thead>
<tr>
<th>Scores</th>
<th>Mean Control</th>
<th>Mean Experimental</th>
<th>Std. Dev Control</th>
<th>Std. Dev Experimental</th>
<th>N Control</th>
<th>N Experimental</th>
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<tbody>
<tr>
<td>COMPASS M1</td>
<td>21.71</td>
<td>26.46</td>
<td>2.71</td>
<td>2.06</td>
<td>1613</td>
<td>693</td>
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<tr>
<td>COMPASS M2</td>
<td>18.63</td>
<td>20.45</td>
<td>4.16</td>
<td>4.80</td>
<td>1613</td>
<td>693</td>
</tr>
</tbody>
</table>
### Table 5: Logistic Regression Output for Passing Arithmetic on First Enrollment

| Did Pass Arithmetic | Coef.  | Std. Err. | z      | P>|z| | [95% Conf. Interval] |
|---------------------|--------|-----------|--------|-----|----------------------|
| initials1 score     | .1251368 | .017554  | 7.13 | 0.000 | .0907316  | .1595419 |
| initials2 score     | .0584985 | .0110389 | 5.30 | 0.000 | .0368625  | .0801344 |
| 1.course1_en        | .5467034 | .1301678 | 4.20 | 0.000 | .2915792  | .8018277 |
| _cons               | -3.859068 | .4188744 | -9.21 | 0.000 | -4.680047 | -3.038089 |

Numbers of obs = 2306  
LR chi2 (3) = 247.55  
Prob > chi2 = 0.0000  
Pseudo R2 = 0.0785  
Log likelihood = −1452.6269

### Table 6: Cross Tabulation Output for Passing Arithmetic on First Enrollment

<table>
<thead>
<tr>
<th>course1_en</th>
<th>Did Pass First Semester</th>
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<tr>
<td></td>
<td>No</td>
</tr>
<tr>
<td>MA005</td>
<td>901</td>
</tr>
<tr>
<td>MA005M</td>
<td>189</td>
</tr>
<tr>
<td>Total</td>
<td>1,090</td>
</tr>
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</table>

Fisher’s exact = 0.000  
1-sided Fisher’s exact = 0.000

### Table 7: Logistic Regression Output for Testing at Least Once

| Did Test | Coef.  | Std. Err. | z      | P>|z| | [95% Conf. Interval] |
|----------|--------|-----------|--------|-----|----------------------|
| initials1 score | .0399709 | .0181139 | 2.21 | 0.027 | .0044682  | .0754735 |
| initials2 score | .0295013 | .0115562 | 2.55 | 0.011 | .0068516  | .0521509 |
| 1.course1_en   | .7794594 | .1435288 | 5.43 | 0.000 | .4981482  | 1.060771 |
| _cons          | -.7949681 | .4262577 | -1.86 | 0.062 | -1.630418 | .0404816 |

Numbers of obs = 2306  
LR chi2 (3) = 101.20  
Prob > chi2 = 0.0000  
Pseudo R2 = 0.0362  
Log likelihood = −1346.1622
### Table 8: Cross Tabulation Output for Testing at Least Once

<table>
<thead>
<tr>
<th>course1_en</th>
<th>Did Test</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Did Test</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Did Test</td>
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<td></td>
</tr>
<tr>
<td>MA005</td>
<td></td>
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<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Fisher’s exact = 0.000  
1-sided Fisher’s exact = 0.000

### Table 9: Cross Tabulation of Enrollment in Algebra

<table>
<thead>
<tr>
<th>course1_en</th>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Did Enroll in Algebra</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Did Enroll in Algebra</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MA005</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MA005M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fisher’s exact = 0.000  
1-sided Fisher’s exact = 0.000

### Table 10: Cross Tabulation of Passing Algebra

<table>
<thead>
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<tbody>
<tr>
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<td>Did Enroll in Algebra</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Did Enroll in Algebra</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MA005</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MA005M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
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</table>

Fisher’s exact = 0.062  
1-sided Fisher’s exact = 0.035

### Table 11: Cross Tabulation of Passing Algebra (Fall 2009 Students Only)

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<tbody>
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<td></td>
</tr>
<tr>
<td></td>
<td>Did Enroll in Algebra</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MA005</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>MA005M</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fisher’s exact = 0.581  
1-sided Fisher’s exact = 0.311
References


The Effects of Study Skills Training and Peer Coaching of ‘At-Risk Students’ on Retention and Passing Rates in a Remedial Mathematics Course

Associate Professor Leonid Khazanov, Mathematics Department
Assistant Professor Michael George, Mathematics Department
Assistant Professor Chris McCarthy, Mathematics Department

Borough of Manhattan Community College
Summary of Background and Method

The aim of this project was to implement a multifaceted and differentiated approach to improving passing rates and reducing attrition rates in Elementary Algebra in a community college setting. Our approach involved:

• Incorporating the teaching of study skills, time management strategies, test-taking skills, and anxiety reduction strategies into the Elementary Algebra course.
• Identifying students at high-risk of failure or dropout at the beginning of the semester.
• Assigning peer coaches to high-risk students to help them pass the course.
• Extensive training of instructors and coaches.

Our specific objective was to implement and test an intervention model that would affect the maximum possible number of students in each participating section. Our research hypothesis was that incorporating study skills training and anxiety reduction strategies in the Elementary Algebra course and providing additional counseling, peer mentoring, and tutoring for at-risk students would result in improved student persistence and higher passing rates in Elementary Algebra.

The independent variables in our study were:

• Study skills training
• Test-taking skills training
• Time management skills training
• Math anxiety reduction
• Test anxiety reduction
• Tutoring and mentoring (coaching) to reinforce skills taught in class and provide personalized assistance to at-risk students

The dependent variables were:

• Course pass rates
• Student retention rates in the course
• End-of-semester pass rates

Background and Significance

Research has documented that only about 25% of the variation in students’ performance is attributable to the quality of teaching of the subject matter; another 25% is explained by affective variables such as attitudes, study habits and skills, dispositions, and math and test anxiety (Bloom, 1976; Nolting, 2008), with the remaining 50% attributable to cognitive entry skills (aptitude and prerequisite knowledge of the subject). According to Nolting, students’ affective characteristics are the most neglected area in colleges today.

Our challenge was to find an effective way of reaching out to all students, including high-risk students. We believed that teaching study skills in the classroom only is not enough to make a difference for all students. At-risk students need additional support; they might benefit from having a mentor, a personal tutor, and someone who will monitor their attendance, time management, homework completion, acquisition of study skills, and preparation for examinations. This additional assistance was offered by trained peer coaches.
Method

PARTICIPANTS AND SETTING

Participating students were BMCC students taking MAT 051, Elementary Algebra, in the Fall 2010 semester. MAT 051 is a remedial, non-credit course. Five instructors were randomly selected to participate. One section taught by each instructor was randomly assigned to the experimental condition and the other section assigned to the control condition.

To identify at-risk students three instruments were used: a survey, a diagnostic arithmetic test, and instructor observations. The survey was designed to assess a number of risk factors, including academic background, motivation, study skills, and math and test anxiety. The diagnostic arithmetic test was comprised of ten multiple-choice problems covering such topics as whole numbers, fractions, decimals, and percents. Instructor observations included class attendance, timely report to class, homework completion, and the extent of participation in class activities. Depending on their performance on the above instruments students were awarded risk points. Each instrument carried a maximum of two risk points. Consequently, students’ aggregate risk score could vary from zero to six, six being the highest possible risk score. Students with the risk score of three or higher were classified as at-risk of course failure and were offered coaches.

INDEPENDENT AND DEPENDENT VARIABLES

The independent variables in the study included the intervention administered in the experimental classroom, consisting of study skills training, time management instruction, and math and test anxiety desensitization. These were integrated with the elementary algebra instruction throughout the course. Within these sections, students identified as “at-risk” received supplementary coaching sessions that reinforced these skills through tutoring and mentoring.

The dependent variables were the end-of-semester pass rates, course pass rates, and student retention in the course.

DESIGN AND PROCEDURE

Participating faculty attended a pre-semester training workshop delivered by a study-skills and a time management expert. Coaches attended a training session including a case study analysis of how to work with students.

Coaches were instructed to meet with their protégés at least once a week for a minimum of one hour to discuss progress, review homework, and help prepare for tests and quizzes. Coach duties also involved assisting their protégés in building strong study skills and developing effective time management strategies. Coaches were compensated for their activities.

Findings

The data shows that (1) the retention rate was significantly higher in the treatment groups \( p = 0.01299, \) see Figure 1); (2) the passing rate was higher for the treatment groups, but not significantly so \( p = 0.3013, \) see Table 1); (3) our diagnostic test successfully identified at-risk students \( p = 0.0021, \) see Figure 2; (4) the overall passing rate for coached students was significantly higher than for those who were not assigned coaches when adjusted for risk scores \( p = 0.03356, \) see Figure 3).
Note: the treatment sections achieved an overall 36.4% passing rate versus the 32.8% passing rate of the control sections. Also note that the overall attrition (withdrawal) rates for the treatment sections (13.2%) is lower than the attrition rate for the control sections (25.0%).

**Suggestion for Related Research**

We suggest an alternative model for coaching. In particular, instead of coaches working with protégées one-on-one, students will be organized into heterogeneous groups, not limited to at-risk students. It appears that some lower-risk students could also benefit from a relationship with a coach. Some students in the experimental sections asked for a coach but could not be awarded one due to their risk point score, and some of these students went on to fail the course. Some at-risk students might also benefit from working with more successful students in the class.

**Figure 1:** Retention Proportions

- Control Group
- Treatment Group

*p-value = 0.01299 (2x2x5 Exact Test, Combinatorial)*
We also suggest a way to fine-tune the process of matching coaches with protégées and of managing the coach-protégé relationship. Many coaches found their assigned protégées to be unreliable with respect to arranging meetings. In future projects, there should be stricter guidelines on the responsibilities of protégées, and those protégées who do not satisfy the guidelines, i.e., fail to show up for appointments, should be replaced by new protégées.

References


The Gap Project: Closing Gaps in Gateway Mathematics Courses

Assistant Professor Sandra Kingan, Mathematics Department
Assistant Professor Anthony Clement, Mathematics Department
Associate Professor Jun Hu, Mathematics Department

Brooklyn College
Abstract

The Gap Project was an experiment to determine if focused extra coaching and advising delivered at the right time to Precalculus students with gaps in their mathematical background would improve the overall pass rates across all sections. A sample of students attended workshops in August 2010, just before the Fall semester began. In selecting the sample, every effort was made to reach students with a weak grasp of prerequisites. For example, COMPASS scores were used when available, as well as a Diagnostic Test. Subsequently the students took Precalculus with a cross section of instructors. To avoid bias in data, the principal investigator and co-principal investigators did not teach any section. Students who attended any portion of the workshop, were considered treated, so as to ensure student motivation was not a lurking variable. Indeed some of the treated students who received an F grade attended a very small portion of the workshop. At the end of the semester the F/W rate in all sections of Precalculus was 31.49%. The F/W rate for students who participated in the workshop was 22.45%. A large sample significance test for population proportion was used, after confirming that the sample size met the requirements of this test. The $z$-score was -1.36; the $p$-value was 0.0869. This is significant at $\alpha$. Efforts to identify students lacking prerequisite knowledge revealed that nearly a third of the students enrolled in Precalculus either did not take the COMPASS placement exam or took it years ago. The next semester, a Diagnostic Test was prepared and given to students on the first day of class. An effective cut-off score was determined by correlating Diagnostic Test scores with final grades.

Introduction

The Gap Project was an experiment to determine if focused extra coaching and advising delivered at the right time to Precalculus students with gaps in their mathematical background would improve the overall pass rates across all sections. Students who pass the placement exams and are ostensibly ready to take Precalculus have certain gaps in their preparation, regardless of where passing levels on the placement exam are set. The objectives of this project were to determine the gaps in knowledge that exist between high school and college level pre-calculus; to identify the students with these gaps; and to develop and test methods for closing these gaps. The hypothesis was that focused extra coaching delivered at the right time to students lacking prerequisites will improve student learning and pass rates in Precalculus.

The project began with an examination of the gaps in student prerequisite knowledge that most affect student performance in a Precalculus course. Tutors from the Tutoring Center collected data on student difficulties. They reported that students had too much trouble with topics in the very first chapter including linear and absolute value equations and inequalities and difficulties with basic concepts of functions and graphs. Given that the syllabus for Precalculus includes algebra, trigonometry, and conic sections, not knowing material in the first chapter made for an inauspicious beginning and a high withdrawal (W) rate.

Next, the Director of Testing gave a survey to 190 students immediately after they took the COMPASS exam. The True/False questions revealed that the majority of students who took the COMPASS did not know simple pre-algebra concepts. Table 1 lists the questions, each of which is a false statement. Yet many students thought they were true or didn’t respond (see Figure 1).
**Table 1:** Results of the True/False survey

<table>
<thead>
<tr>
<th></th>
<th>True</th>
<th>False (Correct Ans)</th>
<th>Blank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{3} = 0.333333$</td>
<td>141</td>
<td>10</td>
<td>39</td>
</tr>
<tr>
<td>$\frac{1}{0} = 0$</td>
<td>62</td>
<td>89</td>
<td>39</td>
</tr>
<tr>
<td>$\frac{3}{5} + \frac{2}{10} = \frac{3 + 2}{5 + 10} = \frac{5}{15} = \frac{1}{3}$</td>
<td>18</td>
<td>131</td>
<td>41</td>
</tr>
<tr>
<td>$-3^2 = 9$</td>
<td>118</td>
<td>31</td>
<td>41</td>
</tr>
<tr>
<td>$(x + y)^2 = x^2 + y^2$</td>
<td>53</td>
<td>89</td>
<td>48</td>
</tr>
<tr>
<td>$\frac{(x + y)}{x} = 1 + y$</td>
<td>33</td>
<td>106</td>
<td>51</td>
</tr>
<tr>
<td>$\sqrt{x+y} = \sqrt{x} + \sqrt{y}$</td>
<td>84</td>
<td>57</td>
<td>49</td>
</tr>
<tr>
<td>$\frac{x+y}{x} = y$</td>
<td>52</td>
<td>89</td>
<td>49</td>
</tr>
<tr>
<td>$(x + 2) - (2 - x) = -2x$</td>
<td>26</td>
<td>99</td>
<td>65</td>
</tr>
<tr>
<td>$3(2x - 5) = 6x - 5$</td>
<td>12</td>
<td>113</td>
<td>65</td>
</tr>
<tr>
<td>$\frac{1}{x} ÷ (1 - \frac{1}{x}) = x - 1$</td>
<td>41</td>
<td>80</td>
<td>69</td>
</tr>
<tr>
<td>$f(x) = x^2$, then $f(x + h) = x^2 + h$</td>
<td>71</td>
<td>53</td>
<td>66</td>
</tr>
</tbody>
</table>

**Figure 1:** Results of the True/False survey

[Bar chart showing the results of the True/False survey]
The survey also revealed strong evidence for “fear of fractions.” Students were asked to pick which equations/inequalities they considered most difficult to solve. Conceptually the equation with the fraction is the easiest to solve, but 69% identified it as the hardest equation and 78% identified the inequality with a fraction as the hardest among inequalities.

**Figure 2:** Responses to the question “Among the following equations, choose the one you consider most difficult to solve.”

<table>
<thead>
<tr>
<th>Problems</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>3%</td>
</tr>
<tr>
<td>2</td>
<td>127</td>
<td>69%</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2%</td>
</tr>
<tr>
<td>4</td>
<td>47</td>
<td>26%</td>
</tr>
</tbody>
</table>

**Figure 3:** Responses to the question “Among the following inequalities, choose the one you consider most difficult to solve.”

<table>
<thead>
<tr>
<th>Problems</th>
<th>Frequency</th>
<th>Percentage</th>
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<tr>
<td>1</td>
<td>5</td>
<td>3%</td>
</tr>
<tr>
<td>2</td>
<td>144</td>
<td>78%</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>4%</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
<td>16%</td>
</tr>
</tbody>
</table>

**Implementation and Statistical Analysis of Preparation Workshops**

Preparatory Workshops were held in August 2010 for 59 of the 470 students who took Precalculus in Fall 2010. Two types of workshops were offered and different recruitment methods were used. One workshop met for three hours per day for four days. Another met for four hours on 2 days and included online communication before and after the workshop.

For the first two workshops students with a weak grasp of prerequisites were identified based on COMPASS scores and students with low COMPASS scores were invited to participate. Twenty students participated for all or some portions of these workshops. The effort to identify students lacking prerequisite knowledge revealed that nearly 30% of students enrolled in Precalculus did not take the COMPASS. Of those who took it, several took it two or three years ago or longer.
For the third workshop all students enrolled in Precalculus were invited to attend. A simple Diagnostic Test at the beginning of the workshop revealed that the highest score was 66% with only 3 students scoring higher than 50%.

At the end of the semester the F/W rates of treated and untreated students were found (see Table 2). Ten of the treated students did not take Precalculus. They must have changed their schedule before the semester began. The F/W rate for all Precalculus students was 31.49%. The F/W rate for students who took workshops was 22.45%.

<table>
<thead>
<tr>
<th>Table 2: F/W rates of treated and untreated students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
</tr>
<tr>
<td>Treated Students</td>
</tr>
<tr>
<td>Untreated Students</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

A large sample significance test for population proportion was used. The sample size was $n = 49$; the population proportion was $p = 0.3149$; and the sample proportion was $p = 0.2245$. In order to use this test, one must check that $np$ and $n(1 - p)$ are both at least 10 because otherwise the distribution is not normal. In this case $np = 15.43$ and $n(1 - p) = 33.57$. At the same time, the sample cannot be bigger than 10% of the population as a rule-of-thumb or else sampling without replacement becomes an issue. In this case 10% of the population is 47. The sample size was 49, which is barely over 10%. This is small enough to meet the criterion. Thus the sample size was not too large, nor too small and met the two requirements for using this test. The $z$-score was $-1.36$ and the $p$-value was 0.0869. The result was significant at $\alpha = 0.1$. This means the probability of this happening by random is less than 10%. A small value of $\alpha$ between 0 and 1 is desirable.

**Effectiveness of Placement Tests and the Impact of Motivation on Student Performance**

As a result of the low performance on the Diagnostic Test and the lack of evidence to support using the COMPASS exam, the focus shifted to effectiveness of placement tests and student motivation. Conventional wisdom suggest that student success in multi-section lower division service courses is typically due to one of three reasons: lack of motivation, lack of prerequisites, or large variability in quality of instruction. Since all three factors are intertwined, it is difficult to disentangle exactly what is going on in a particular situation.

On the first day of class in Spring 2011, all Precalculus students were given a customized 20 minute Diagnostic Test that checked basic Pre-algebra concepts and a Motivation Test that measured student motivation as a prediction of success. The purpose of the Motivation Test was to determine how important the student thought the test was to them. High positive correlation ($0 < r < 1$) between the Diagnostic Test and the Motivation Test would indicate that motivated students do better. A Motivation Test used was prepared by Motivation Research Institution of James Madison University (http://imri.cisat.jmu.edu/). The validity of this test has been verified. See for example [2].
A total of 176 students took the Diagnostic Test and 159 of these students took the Motivation Test. Mean motivation scores were considerably higher than mean Diagnostic scores (see Table 3). However, as Figure 4 shows, there is no correlation between the Diagnostic Test and the Motivation Test ($n = 159, r = -0.07$). This counter-intuitive outcome indicates that student motivation is not a factor influencing the outcome of the Diagnostic Test. The math scores are low whether or not the students are motivated. Rather than question the validity of the Motivation Test, it is reasonable to conclude that students are motivated, but genuinely do not know or cannot recall the prerequisites at the beginning of the semester.

**Table 3**: Summary of results of the Diagnostic Test and Motivation Test

<table>
<thead>
<tr>
<th></th>
<th>Diagnostic Test Math Scores</th>
<th>Diagnostic Test Motivation Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>176</td>
<td>159</td>
</tr>
<tr>
<td>Mean</td>
<td>43%</td>
<td>66%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>15.23</td>
<td>12.45</td>
</tr>
</tbody>
</table>

**Figure 4**: Scatterplot of math scores versus motivation scores

At the end of the semester a spreadsheet was prepared with scores and final grades. Of the 176 students who took the Diagnostic Test in the beginning of the semester, 9 received ABS/INC grades, bringing the total down to 167. Of these, 16 students did not do the Motivation Test. So the number of data values for Diagnostic Test scores versus final grades was 167 and the number of data values for Motivation Test scores and final grades is 151.

The association between the scores and final grades were examined in two different ways since the scores are quantitative and the final grades are qualitative: with a box plot of grades A, B, C, D, F, W with the x-axis showing grades and the y-axis showing scores; and with a scatterplot of grades versus scores.
In the boxplot, the dark horizontal line in each box represents the median, the ends of the box represent the 75th and 25th quartiles and the tails extend to the highest and lowest scores. Outliers are plotted individually. Figure 5 indicates an association between Diagnostic Test scores and final grades. Students with an A grade had a higher median score than the other grades.

But quite unexpectedly the association is small. Students with high Diagnostic Test scores were getting F. On the other hand, students with low Diagnostic Test scores were getting A, B, and C grades. Based on the data, it would be reasonable to say that a student with a Diagnostic Test score lower than 20% is likely to fail. This analysis is, of course, specific to the Diagnostic Test used, but it does indicate the importance of basing cut-offs on data rather than a standard grading system.

Figure 6 compares Motivation Test scores with final grades. Students with a final grade of A scored higher on the Motivation Test than other students. However, this pattern doesn’t continue. Students with a final grade of C have a higher median motivation score than those with a final grade of B. Moreover, the variability for F and W is high.

**Figure 5:** Association between final grades and Diagnostic Test scores with box plot and scatter plot

**Figure 6:** Association between final grades and Motivation Test scores (0–50)
Conclusions

How does student motivation impact the results of our experiment?
With respect to the Precalculus Preparation Workshops and the hypothesis that students who attend preparation workshops do better in the course, students who attended one day out of a four day workshop, two hours out of a six hours workshop etc., were considered “treated.” It is difficult to conclude that students who cannot even attend the full workshop are motivated. Nevertheless, they were considered treated and their F grades were included in the outcome. Even so, the F/W rate of those who attended the workshops is only 22.45%. While this number may seem high, it is significantly better than the F/W rate of the untreated students which was 32.54%. (The z-score is −1.36 and the p-value is 0.0869; this is significant at α = 0.1).

How does student motivation impact their performance on the Diagnostic Test?
Diagnostic Test scores and Motivation Test scores appear to be unrelated for the 161 students who took the Diagnostic Test and the Motivation Test. Regardless of motivation, students do not have the prerequisites at the beginning of the course.

How effective is a Diagnostic Test at predicting final grades in Precalculus?
The Diagnostic Test was designed to be flexible, allowing for minor changes from semester to semester, such as scrambling the True/False questions and changing the equations to be solved. An analysis of the data revealed that it is effective in predicting final grades, but the association is small. The cut-off turned out to be 20%. In other words, students taking this Diagnostic Test should be advised to drop the course if their score is below 20%. Colleges and departments preparing their own Diagnostic Test will find a different cut-off. The key is to set cut-offs by gathering and analyzing data instead of a conventional grading system.

What are the implications for the COMPASS Test?
The case-study on placement testing revealed no evidence to support using the COMPASS as a placement test. Nearly a third of the students enrolled in Precalculus either did not take the COMPASS test or took it years ago. If the COMPASS score was obtained several years ago, it is meaningless. If the exemption is due to good grades in high school courses, then the quality of the high school becomes a consideration. Basing placement on test scores under these circumstances raises questions of equity.

How important are prerequisites for Precalculus?
Overall, this project revealed the difficulties in determining prerequisite knowledge for Precalculus. The emphasis should be less on what the students did in the past and more on what students will do during the semester, starting with a solid preparation workshop just before classes begin, where they are advised of what to expect in terms of time commitments and study expectations. This may be followed by an in-house Diagnostic Test given to all students taking Precalculus either just before or on the first day of class. The cut-off score should be based on an analysis of Diagnostic Test scores and final grades from previous semesters.

References
Teaching Pre-Algebra and Algebra Concepts to Community College Students through the Use of Virtual Manipulatives

Associate Professor Violeta Menil, Mathematics Department
Hostos Community College

Adjunct Lecturer Eric Fuchs, Department of Mathematics and Computer Science
Bronx Community College
Background

This research project’s goal was to investigate the effectiveness of using virtual “manipulatives” on the learning of basic mathematical concepts by community college students enrolled in pre-algebra and algebra remedial classes. The impact on students’ attitudes toward mathematics and their confidence in doing mathematics were also investigated.

Manipulatives allow students to build up mental representations and acquire skills in using and modifying these representations and synthesizing new ones (Davis 1984; Kilpatrick 1985; Thompson, 1982). They have been described and found by many to be the best approach to resolve the difficulties inherent in learning arithmetic and algebra concepts and processes (Boas, 1981; Bruner, 1966; Davis, 1988; Dienes, 1971; Gningue, 2000; Kaput, Carraher, & Blanton, 2008; Macgregor & Stacey, 1997; Sobol, 1998).

Whereas hands-on manipulatives are tactile and visual, virtual manipulatives are only visual; on the other hand, virtual manipulatives are interactive: that is, the learner can manipulate the same objects and create the same mental representations of the objects using the computer mouse (http://nlvm.usu.edu). In today’s technology-enriched classrooms, it is even more appealing for college professors and college students to use computers rather than hands-on manipulatives.

Bruner asserted that learning by discovery involved an internal reorganization of previously known ideas and stipulated that children move through three modes or levels of representation as they learn (Bruner, 1966). In the first or enactive level, the child needs action on materials to understand a concept. In the second or iconic level, the child creates mental representations of the objects but does not manipulate them directly; rather, the concept is represented pictorially. Finally, in the third or symbolic level, the child is strictly manipulating symbols and does not need to manipulate objects.

On the continuum from concrete to abstract representation of mathematical concepts, virtual manipulatives fall between the enactive level and iconic level: the student handles the squares, blocks and fraction pieces using the mouse and computer screen. In this research, we assumed, based on our experience working with colleagues and students, that college students might be reluctant (or offended) if asked to learn mathematics concepts using physical manipulatives, as children do.

Research Design and Method

Two CUNY colleges were involved in this research project during the Fall, 2010, Spring 2010 and Spring, 2011 semesters: Hostos Community College and Bronx Community College (BCC). The two investigators (Menil from Hostos and Fuchs from BCC) designed modules using virtual manipulatives related to the study of pre-algebra (integers, fractions, decimals, ratios, and percents) and algebra (polynomial operations, factoring, functions and equation solving), concepts that constitute more than 50% of the mathematics curriculum. They taught the control groups and experimental groups in their respective campuses. Two sections of pre-algebra and two sections of algebra classes were examined. The four classes were assigned to the two investigators by their respective department chairs as part of the CUNY system of faculty teaching load. The experimental groups primarily learned the pre-algebra and algebra concepts—using the technology on virtual manipulatives while the control groups learned the same concepts through—the traditional lecture-style pedagogy. A one-way Analysis of variance (ANOVA) was used to investigate students’ performance on pre-algebra and algebra concepts with and without the use of virtual manipulatives. The study used the technology on virtual manipulatives as the independent variable. The students’ scores in the different assessments in
both the pre-algebra and algebra were the dependent variables. Results of the Fennema-Sherman Mathematics Attitude Scales were a part of the qualitative analysis of students’ attitudes toward mathematics and confidence in doing mathematics.

This research study was quasi-experimental. The sample, represented by students in the two gateway courses, pre-algebra and algebra, was of convenience since the investigators taught these courses; however, the classes selected for intervention (technology-virtual manipulatives) and control (lecture) were randomly selected by the students who, upon registering, were not aware of this research project, let alone knowing whether they would be part of an experimental or a control group.

On the first day of classes, the students received a detailed explanation of the purpose of the project. The two investigators explained the content of the IRB consent form before they signed it. The students were given the option of not participating in the project and switching to a different section. In both colleges, all the students agreed to participate in the research project and to stay in their respective group. None of the students elected to switch sections, neither at Hostos nor at BCC.

When signing the consent forms the students in the experimental groups were given the option not to be audio recorded, not to be video recorded, or both. The investigators honored the students’ elections throughout the project.

**Data Collection**

**QUALITATIVE DATA COLLECTED**

- Students provided responses to a 24-question survey on attitudes toward mathematics and confidence in doing mathematics. The questionnaires were based on two subscales of a revised version of the Fennema-Sherman Mathematics Attitude Scales (Hackett & Betz, 1989). Identical questionnaires were given to the students at the beginning and end of each class. As a certain percentage of the students (approximately 20%) dropped out of the classes before conclusion, not all students who filled the questionnaires at the beginning of the class filled questionnaire at the end of the class.

- The classes were audio/video recorded, sometimes in their entirety, sometimes for only part of the class. We put more emphasis on video recording of students’ work with the virtual manipulatives in the experimental classes than on students’ work in the control groups, since teaching in the control group classes did not differ from the traditional teaching used in other sections in the two colleges. The video recording was done by the research assistants, or in the case of BCC by the lab technician as well. Some classes were video recorded in their entirety by using one (or two) video recorders.

- The recorded events were analyzed by the investigators, who sought patterns of learning and compared the learning with virtual manipulatives to the learning in lecture-format classrooms. The use of students as co-teachers in the technology-infused classroom was documented through video recordings as well. The investigators produced several video clips illustrating the learning by the students who used virtual manipulatives.

- Several students wrote weekly reflections that provided investigators with insight into their mathematical difficulties and attitudes toward use of technology in learning mathematics. Some students also produced mathematical autobiographies that provided the investigators with insight into the students’ home culture and exposure to learning of mathematics in their elementary, middle and high school years.
QUANTITATIVE DATA COLLECTED IN THE PRE-ALGEBRA CLASSES

- Students’ responses to the initial pre-algebra test (composed by the two investigators) consisting of 25 questions covering the following topics: operations with whole numbers, operations with fractions, operations with decimals and percentages, ratios and proportions, order of operations, operations with signed numbers, and related word problems. This test was administered during the first week of the pre-algebra classes.

- Students’ responses to a pre-final test consisting of the same 25 questions that appeared in the initial test mentioned above. This test was administered during the last week of the pre-algebra classes and provided an indication of the individual student’s learning as well as the understanding of the entire class.

- Students’ scores in the achievement tests at the conclusion of the units dealing with integers; fractions and decimals; ratio and proportion; percents; uniform pre-algebra Hostos and BCC final examinations (different examinations were used by the respective mathematics department at the two colleges).

- Students’ scores in the COMPASS exit test. These data were not compiled in the production of this report, since the two colleges have different criteria for allowing students to take the COMPASS as an exit exam:
  - At Hostos only students who passed the departmental mid-term examination were allowed to participate in the COMPASS exit exam. This policy is strictly enforced. At BCC all students who participated in this project were allowed to take the COMPASS exit exam.
  - There is a difference of policy between the two colleges in what constitutes the passing grade on the COMPASS test. The requirements at Hostos are stricter than the requirements at BCC.
  - CUNY decided that effective January 2011 the COMPASS test is going to be used only as an entrance exam and for course placement purposes, but not as an exit exam for evaluating student learning.

QUANTITATIVE DATA COLLECTED IN THE ALGEBRA CLASSES

- Students’ scores in the achievement tests at the conclusion of the units dealing with polynomial operations; factoring; functions and solving linear equations.

- Students’ scores in the uniform algebra Hostos and BCC final examinations (different examinations were used by the respective mathematics department at the two colleges).

- Students’ scores in the COMPASS exit exam. These data were not compiled for producing this report, for the reasons explained above.

In this research project, the following hypotheses were tested:

- Due to the intervention (virtual manipulatives) significant differences in the students’ performance in pre-algebra are expected, with experimental group doing better than the control group;

- Due to the intervention (virtual manipulatives) significant differences in the students’ performance in algebra are expected, with experimental group doing better than the control group;
Due to the intervention (virtual manipulatives) there will be a significant improvement in experimental group students’ confidence in their mathematical ability.

Due to the intervention (virtual manipulatives) there will be a significant improvement in experimental group students’ attitude toward mathematics.

Results and Findings

HOSTOS PRE-ALGEBRA

Table 1 shows that for the Hostos pre-algebra classes, the experimental group outperformed significantly the control group in all five assessments: Exam 1 (fractions), Exam 2 (decimals), Exam 3 (percents) Exam 4 (integers), and the Final Examination. The mean differences of the five assessments between the experimental and control groups were all significant, ($p < .05$ and $p < .10$). Please see Table 1 for findings. Overall, the significant differences were in favor of the experimental group.

<table>
<thead>
<tr>
<th>Exam</th>
<th>Mean</th>
<th>SD</th>
<th>n</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exam 1</td>
<td>Experimental</td>
<td>67.7391</td>
<td>18.0384</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>58.3182</td>
<td>17.8069</td>
<td>22</td>
</tr>
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<td>Exam 2</td>
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<td>76.1905</td>
<td>17.8371</td>
<td>21</td>
</tr>
<tr>
<td></td>
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<td>63.4000</td>
<td>14.9961</td>
<td>20</td>
</tr>
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<td>22.1704</td>
<td>21</td>
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<td></td>
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<td>23.0434</td>
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<td>Exam 4</td>
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<td>59.7619</td>
<td>22.2866</td>
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</tr>
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<td>46.0625</td>
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<td>Final</td>
<td>Experimental</td>
<td>60.2125</td>
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</tr>
<tr>
<td></td>
<td>Control</td>
<td>45.3929</td>
<td>23.6521</td>
<td>14</td>
</tr>
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</table>

Figure 1: Hostos Pre-Algebra (Fall 2010): Experimental vs. Control Means on the Five Assessments
Table 2 reveals that for the BCC pre-algebra classes, the mean scores for the control group are higher than that of the experimental group in all the five assessments: Exam 1, Exam 2, Exam 3, Exam 4 and the Final Examination. Notice that Exam 4 has the smallest mean difference in favor of the control group. The graph shows significant mean differences in all the five assessments in favor of the control group, \((p < .001, p < .01, p < .05 \text{ and } p < .10)\). Factors that seem to have contributed to the control group's better performance over the experimental group include time effect (8:00 AM class vs. 10:00 AM); age effect (younger students vs. mature students); student preparation (inadequately prepared vs. better prepared students). See Table 2 for the significant findings in favor of the control group.

Table 2: BCC Pre-Algebra (Fall 2010): Experimental vs. Control Means on the Five Assessments

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>n</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exam 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td>Experimental</td>
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<td>Exam 2</td>
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<td></td>
<td>Experimental</td>
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<td>Exam 3</td>
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<tr>
<td></td>
<td>Experimental</td>
<td>68.1765</td>
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<td>Control</td>
<td>82.2727</td>
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<td></td>
<td>Experimental</td>
<td>65.8125</td>
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</tbody>
</table>

Figure 2: BCC Pre-Algebra (Fall 2010): Experimental vs. Control Means on the Five Assessments
HOSTOS ALGEBRA

Table 3 shows that for the Hostos algebra classes, the mean performance of the experimental group is higher in three of the four assessments: Exam 1, Exam 3 and the Final Exam. However, the mean differences in these three exams between the experimental and control were not all significant. The graph shows that the mean difference in exam 3 is the only assessment that resulted to a significant finding ($p < 0.05$). See Table 3.

Table 3: Hostos Algebra (Spring 2011): Experimental vs. Control Mean Comparisons on the Four Assessments

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>n</th>
<th>Significance</th>
</tr>
</thead>
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<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td>65.9286</td>
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<tr>
<td>Exam 2</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td>69.3571</td>
<td>28.5362</td>
<td>28</td>
<td>N.S.</td>
</tr>
<tr>
<td>Control</td>
<td>79.8333</td>
<td>23.0551</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>Exam 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
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<td>20.2471</td>
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<td>SIG.@.05</td>
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<td>Control</td>
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<td>Final</td>
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</tr>
<tr>
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<td>52.1600</td>
<td>21.7269</td>
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<td>Control</td>
<td>48.1053</td>
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</table>

Figure 3: Hostos Algebra (Spring 2011): Experimental vs. Control Mean Comparisons on the Four Assessments

BCC ALGEBRA

Table 4 shows that for the BCC algebra classes, the experimental group had a higher mean performance than the control group in all the four assessments. However, the mean differences in these four assessments were not all significant. The graph shows that the mean difference in Exam 2 was the only assessment that was significant ($p<0.05$). See Table 4.
Interpretation of Results

PRE-ALGEBRA

At Hostos pre-algebra classes, the experimental group outperformed significantly the control group in all the five assessments, thereby confirming one of our initial hypotheses (p < .05 and p < .10). By contrast, BCC pre-algebra control group performed significantly better than the experimental group in all the five assessments (p < .001, p < .01, p < .05, p < .10). There were several reasons for the weaker performance of the students in the experimental group at BCC:

• The control group students arrived to college better prepared in mathematics than the experimental group students; this was well documented by the initial placement COMPASS test results. By sheer chance, the two groups differed significantly in their preparation for college work.

• The experimental group started at 8:00am; many students arrived late to class or lab. Some single mothers had difficulty arranging for childcare in the early morning hours. Several single men worked night jobs and arrived to school late.

Table 4: BCC Algebra (Spring 2010 and Spring 2011): Experimental vs. Control Mean Comparisons on the Four Assessments

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
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<th>n</th>
<th>Significance</th>
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</tr>
<tr>
<td>Exam 2</td>
<td>Experimental</td>
<td>71.3684</td>
<td>25.2019</td>
<td>19</td>
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<tr>
<td></td>
<td>Control</td>
<td>56.0909</td>
<td>20.1823</td>
<td>22</td>
</tr>
<tr>
<td>Exam 3</td>
<td>Experimental</td>
<td>69.7895</td>
<td>25.8963</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>60.5500</td>
<td>19.4733</td>
<td>20</td>
</tr>
<tr>
<td>Final</td>
<td>Experimental</td>
<td>67.3500</td>
<td>24.6177</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>60.0000</td>
<td>18.9816</td>
<td>21</td>
</tr>
</tbody>
</table>

Figure 4: BCC Algebra (Spring 2010 and Spring 2011): Experimental vs. Control Mean Comparisons on the Four Assessments

![Graph showing experimental vs. control mean comparisons on the four assessments.](image)
• The control group students displayed a more mature attitude toward learning mathematics than the experimental students.

**ALGEBRA**

• At Hostos algebra classes, the mean scores of the experimental group were higher than the control group in three of the four assessments. However, the mean differences in these three assessments were not all significant. It was only significant in exam 3, \( p < .05 \), thereby confirming one of our initial hypotheses. In the other campus at the BCC algebra classes, the mean scores of the experimental group were higher than the control group in all the four assessments. However, the mean differences in these four assessments were not all significant. It was only significant in exam 2, \( p < .05 \), thereby confirming one of our initial hypotheses, that due to the intervention (virtual manipulatives) there will be a significant difference in students’ performance in these two gateway courses: pre-algebra and algebra.

**Students’ Attitudes and Confidence**

From the pre-survey to the post-survey, the overall response mean in attitude and confidence significantly changed for only one of the eight groups: the BCC algebra control group \( p < .05 \). When all questions were studied individually however, an increase in the mean responses was noted for all groups for each one of the questions. For two of the “confidence” questions, “I am no good at math,” and “Math has always been my worst subject,” a significant change \( p < .05 \) was found in both experimental pre-algebra classes at BCC and Hostos. This result leads us to believe that the confidence in doing mathematics level of students in the Pre-Algebra Experimental groups’ grew more significantly after using the virtual manipulatives than the confidence level of students in the pre-algebra control groups’ at BCC and Hostos after being taught traditionally.

For the algebra groups, no significant changes were noted from the pre- to post-surveys on any of the attitudes and confidence indicators. The control groups at both BCC and Hostos displayed however, a more positive attitude and had a higher confidence level than any of the other groups at the beginning of the algebra courses. Since algebra students have already passed pre-algebra, they perhaps believe more in their ability to do math than pre-algebra students.

**Effects of Technology**

The infusion of technology in the experimental groups promoted group work and facilitated co-teaching. In these classes students volunteered to be assistant teachers and to help other students. Altogether, these classes were more student-centered. Students learned and practiced many concepts on their own, not needing the teacher to come verify their answers. In the tedious work of subtraction of integers, for instance, the concept of zero-pair is essential to understanding the meaning of subtraction. The technology enabled students to try different combinations and strategies on what type of zero-pair to add, and check the results immediately to validate their answers. In instances when the technology was not available, students struggled to understand the concepts taught while waiting for better teacher explanation. The learning with technology allowed the more conscientious students to repeat the lessons at home, on their computer, until they mastered the concepts. The computer software provided the students with many new practice exercises and instant feedback.
Conclusions and Recommendations

We obtained mixed results as far as the quantitative data were concerned. The qualitative data, however, confirmed our hypotheses that virtual manipulatives were useful for students learning basic mathematics concepts. Students expressed this usefulness clearly in their reflections, in face-to-face interviews and through their answers in the questionnaires on attitudes toward mathematics and confidence in doing mathematics.

The experimental group students overcame their initial mathematics misconceptions with less difficulty than the students in the control group. They found the classes with virtual manipulatives more exciting than traditionally taught mathematics classes. The virtual manipulatives appear to be more useful in teaching pre-algebra remedial course than an algebra remedial course. The reasons are as follows:

- The modules for pre-algebra cover most topics: fractions, decimals, percentages, ratios and proportions, operations with integers and solving linear equations;
- The modules for algebra do not cover inequalities, factoring of polynomials and solving quadratic equations;
- The algebra curricula at both Hostos and BCC are accelerated. Teaching a topic with virtual manipulatives takes longer than teaching the same topic in a traditional lecture-style class. Consequently, learning of algebra with virtual manipulatives require more hours than a typical one-semester class;
- By its nature, algebra is more abstract than pre-algebra. Students learning algebra concepts are not helped as much as students learning pre-algebra with virtual manipulatives

Future studies are needed to determine the longer-term utility of learning remedial mathematics with virtual manipulatives. We could hypothesize that students who learned and mastered mathematics concepts with virtual manipulatives will retain the concepts longer. Should virtual manipulatives be an integral part of teaching remedial mathematics in CUNY? To answer this question we have to consider the investment in technology, the training of faculty and the willingness of the faculty to embrace technology in teaching. In order to maximize the effectiveness of using virtual manipulatives, we recommend the following:

1. Provide an intensive two-week workshop series using virtual manipulatives to those students who failed the pre-algebra COMPASS placement test and those students who are multiple repeaters of the remedial pre-algebra course;

2. Provide training in the use of technology to faculty members who will be conducting those workshops.

We plan to disseminate our findings in seminars relating to developmental mathematics. We also plan to publish our findings in a peer-reviewed journal.
References


Gningue, S. M. (2000). The use of manipulatives in middle school algebra: (Committee on Prospering in the Global Economy of the 21st Century, 2006). This conclusion is consistent with the convincing evidence provided by the National Academies in its report, Rising above the Gathering Storm: Energizing and Employing America for a Brighter Economic Future.


Improving Mathematics Learning at LaGuardia

Professor Frank Wang, Department of Mathematics, Engineering and Computer Science
Professor Prabha Betne, Department of Mathematics, Engineering and Computer Science
Associate Professor Marina Dedlovskaya, Department of Mathematics, Engineering and Computer Science
Professor Joyce Zaritsky, Communication Skills Department

LaGuardia Community College
Abstract

In the spring semester of 2011, LaGuardia Community College undertook a CUNY-sponsored project to study the effect of highly trained Academic Peer Instruction (API) tutors on remedial math students’ use of online material called “EDUCO” and the course outcome. The overall pass rate for the 625 students in the API group is 58.9%, and that for the 415 students in the control group is 56.6%. In terms of grades C- or above, the API group outperformed the control group by 5.2% (33.6% vs. 28.4%). The EDUCO average online tutorial time for the API group is 5h 46m, compared with 3h 12m for the control group. The research hypothesis that API tutors will motivate students to spend more time on studying, which in turn will improve their academic performance appears to be validated. We analyzed the scores of four uniform departmental exams, and the raw final score recorded by EDUCO. Although with this sample size we were unable to reject the null hypothesis, we found that the API sections consistently show better outcomes, e.g. higher pass rates, higher mean exam scores, and lower standard deviation, compared to the control sections. Both faculty and students highly appreciated the intervention, and were impressed by the quality of API tutors. However, surveys revealed that external factors such as family and work responsibilities, as well as student indifference, remain the greatest obstacles to students’ success.

Brief Summary of the Background and Method

At LaGuardia Community College, all sections of MAT 096 Elementary Algebra (the second of the two basic skills math courses) use EDUCO textbook and its online support, which has three major components: (1) tutorials; (2) homework, and (3) quizzes. EDUCO online system is also used for administering departmental midterm and final exams. While it is intuitively plausible that students’ time-on-task for online material is associated with positive changes in learning outcome, it remains unclear how students actually utilize online materials outside the classroom. In our research, we incorporated the highly effective Academic Peer Instruction (API) program, previously employed primarily for credit-bearing courses, into our MAT 096 course.

API is a peer tutoring program based on Supplemental Instruction, a nationally and internationally recognized program. API provides regularly-scheduled group study sessions for all students in the targeted courses. By encouraging all students to participate, even those already doing well, API removes the psychological stigma students feel when they are told to go for tutoring because they are failing. Since 1993, API has demonstrated at LaGuardia that students who participate in API have earned grades that are, on average, one half to one letter-grade higher than those who did not participate. The quality of peer tutors is assured through (a) a careful recruitment process, (b) a well-designed training regimen, and (c) the support and reinforcement provided by weekly meetings between Professor Joyce Zaritsky, the faculty director of API, Andi Toce, the assistant director, and all the peer tutors.

Our hypothesis is that highly qualified API tutors will motivate students to spend more time on studying MAT 096 and use EDUCO online material more effectively, which in turn will improve their academic performance. In the spring semester of 2011, 24 out of 49 sections of MAT 096 were randomly assigned an API tutor. Of the remaining 25 sections, 15 are considered as a control group. The other sections were part of other studies.

We rely on several sources of data to evaluate the API tutoring program. The EDUCO Company has provided us with data for time-on-task for online tutorials. We also examine mean scores on three departmental exams and the raw final grade distribution in the course. LaGuardia’s Institutional
Research Office (IR) has performed an analysis of pass rates and grade distribution. Routine data gathered as part of the API program include weekly student attendance at API tutoring sessions, end-of-semester faculty survey, and end-of-semester student evaluation of the API. We have designed a special questionnaire for the API-EDUCO group.

Data Analysis

We employed multiple assessment tools to ensure that our results are scientifically sound and statistically meaningful. For readers who are less concerned with the technical details, they can skip this section and learn the main points in the next section, Summary of the Main Findings.

IR DATA

Based on the results provided by IR, total enrollment in the 24 sections of API group is 625, and that in the 15 sections of control group is 415. In terms of overall pass rate, the API group outperformed the control group by 2.3% (58.9% vs 56.6%). Many studies suggest that grades C− or above represents a better indicator for students’ long-term success, and we found that the API group outperformed the control group by 5.2% (33.6% vs 28.4%) using that standard of success. Improving retention is one of the goals for our study, and the API group indeed has lower dropout rates than the control group. Table 1 gives the details.

<table>
<thead>
<tr>
<th></th>
<th>API group</th>
<th>Control group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>625 students (24 sections)</td>
<td>415 students (15 sections)</td>
</tr>
<tr>
<td>Overall pass rate</td>
<td>368 students, or 58.9%</td>
<td>235 students, or 56.6%</td>
</tr>
<tr>
<td>C− or better</td>
<td>210 students, or 33.6%</td>
<td>118 students, or 28.4%</td>
</tr>
<tr>
<td>W, WA, WN</td>
<td>29 students, or 4.4%</td>
<td>23 students, or 5.3%</td>
</tr>
<tr>
<td>WU</td>
<td>56 students, or 9.0%</td>
<td>44 students, or 10.6%</td>
</tr>
</tbody>
</table>

A chi square test of association between students’ end of semester status (pass or not pass) and treatment type (API or Control) was found to be not significant (p-value of 0.471). However, a chi square test of association between grade (C− or better, lower than C−) and treatment type (API or Control) reveals a p-value of 0.079 ($\chi^2 = 3.083$, 1 degree of freedom). This result is suggestive that sections with API’s tend to achieve better grades than those without API’s.

EDUCO DATA

The main research hypothesis is that highly qualified API tutors will motivate students to spend more time on EDUCO online material, which in turn will improve their academic performance. Based on data provided by EDUCO Company, the “average tutorial time” for the 630 students in the API group is 5 hours and 46 minutes, compared to 3 hours and 12 minutes for the 427 students in the control group. Together with the IR report, our results appear to confirm our hypothesis.

However, there are a lot of subtleties, as a careful reader might have noticed a discrepancy in the number of students between the IR and EDUCO data. We need to mention that once a student registers in the EDUCO system as a member of a certain section, the student’s score in every assignment, test and exam are recorded and is part of the EDUCO report. Also, the student’s record is retained even if he or
she drops the course. Thus, we need to keep in mind that the “average tutorial time” and other exam means are skewed by non-attending students.

To obtain a more meaningful analysis, individual student data are extracted from the EDU CO and analyzed using the SPSS software. Any student whose score is 0 in all the departmental exams is dropped from the analysis, since such students are most likely to have dropped from the course in the beginning of the semester. With these considerations, there were a total of 586 students in 24 API sections, and 367 students in the 15 control sections. The descriptive statistics are as follows.

Table 2: Descriptive Statistics—API

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exam 1-score</td>
<td>586</td>
<td>.00</td>
<td>100.00</td>
<td>58.4868</td>
<td>19.24067</td>
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<tr>
<td>Exam 2-score</td>
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<td>.00</td>
<td>100.00</td>
<td>61.1492</td>
<td>25.58127</td>
</tr>
<tr>
<td>Final part 1-score</td>
<td>586</td>
<td>.00</td>
<td>100.00</td>
<td>51.8656</td>
<td>28.63735</td>
</tr>
<tr>
<td>Final part 2-score</td>
<td>586</td>
<td>.00</td>
<td>120.00</td>
<td>44.9714</td>
<td>29.37663</td>
</tr>
<tr>
<td>Final Part 1&amp;2 combined</td>
<td>586</td>
<td>.00</td>
<td>100.00</td>
<td>48.4185</td>
<td>27.29872</td>
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<tr>
<td>Valid N (listwise)</td>
<td>586</td>
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<td></td>
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</table>

Table 3: Descriptive Statistics—Control

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exam 1-score</td>
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<td>100.00</td>
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</tr>
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<td>Exam 2-score</td>
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<td>Final part 1-score</td>
<td>367</td>
<td>.00</td>
<td>100.00</td>
<td>50.2269</td>
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<tr>
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<td>367</td>
<td>.00</td>
<td>120.00</td>
<td>45.0863</td>
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<tr>
<td>Final Part 1&amp;2 combined</td>
<td>367</td>
<td>.00</td>
<td>100.00</td>
<td>47.6566</td>
<td>29.69714</td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>367</td>
<td></td>
<td></td>
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</tbody>
</table>

Figure 1:

![Graph showing exam scores for Control and API sections](image-url)
In addition to the above bar chart showing the slight improvement in the API group over the control group, two representative box plots can be found in Section III on page 5. Although with this sample size we were unable to reject the null hypothesis (at $\alpha = 0.05$), it is very encouraging to see that the API sections consistently show better outcomes (higher pass rates and mean exam scores and lower standard deviation) compared to the control sections.

**FACULTY SURVEY**

The end-of-semester faculty survey revealed that participating instructors were overwhelmingly enthusiastic about the idea of using API’s to promote EDUCO usage, and were greatly impressed by the quality of the API tutors. Twenty-one out of twenty-four instructors returned the survey. Seventeen instructors considered that the API tutors had been extremely helpful in terms of improving students’ performance, and the same number of instructors found the quality of API to be excellent. There is only one instructor who reported a negative experience. Selected text responses are the following. “The API program promoted a motivated learning environment among the students. The energy was sustained even over more challenging topics.” “Students feel that they receive more attention with a tutor.” Commenting on their individual API, one instructor wrote: “[The API tutor] was a wonderful role model and very helpful during and outside the classroom.” Another wrote: “[The API tutor] is kind to students with no superior attitude.”

When asked about whether an API tutor makes a difference, 9 instructors reported “Strongly agree,” and 11 reported “Agree.” When asked about whether API improves students’ proficiency using EDUCO, 10 instructors reported “Strongly agree,” and 10 reported “Agree.” One representative comment reads: “My API tutor provided extra support for my students however because of their course load + work load they could not always make full use of this help.”

One question for instructors is “To the best of your knowledge, how many students might have dropped out without our API-EDUCO project?” Eleven chose “Unable to estimate.” One instructor wrote: “API encourages students to stay in class.” Another wrote: “My insight of those students that dropped may be due to the intense course content, their lack of confidence and weak background skills.”

**STUDENT FEEDBACK AND OUTCOME**

Student feedback was, overall, very positive. Some representative comments include “It was great to have a tutor in the class together with the teacher,” “If I had a problem I could call [Mr. X] or send him an email and get a speedy response. His sessions were very helpful and important to me.” “Attending API sessions has helped me to pass MAT 096.”

The most common issue that students raised was scheduling, as some students indicated that their family and job responsibilities prevented them from attending the API sessions. However, most students acknowledged that API tutors have been doing their best to accommodate students’ various needs.

Consistent with the historical API records, the grade difference (using the GPA formula) between students who attended 3 or more sessions (modeled after the nationally recognized Supplemental Instruction program) and students who attended 2 or less sessions is $+0.74$. But this gain is uneven among sections, as the standard deviation is 0.79.
Summary of the Main Findings

In summary, our findings provide preliminary evidence that the API program holds promise. Perhaps the most important metric is the pass rate; the result is 58.9% for the API group, compared with 56.6% for the control group. It is increasingly common among higher education institutions to use grade C− or above as a measure of “success rate.” With this definition, the API group outperformed the control group by 5.2% (33.6% − 28.4%). A chi square test of association between grade (C− or better, lower than C−) and treatment type (API, Control) gives a \( p \)-value of 0.079, which is suggestive that the API sections tend to achieve better grades.

As described in the preceding section, students who unofficially withdraw could greatly skew the means. This situation makes it hard to interpret the “average tutorial time.” In any case, the contrast between the API group (5h 46m) and the control group (3h 12m) is striking. With the sample size of the current study, we were unable to reject the null hypothesis (that the positive outcome is due to random fluctuation). However, when we compare the means of Exam 1, Exam 2, and Final Exam, the API sections consistently show better outcomes; see bar chart in subsection II (b). We also produced box plots for all the departmental exams, and again the API sections consistently show better outcomes. (Below we display the box plots, for overall EDUCO score on the left, and for the Final part 1 and 2 combined score on the right.)

![Figure 2: Overall EDUCO score](image1.png)

![Figure 3: Final part 1&2 combined score](image2.png)

An inherent problem with studies of small effects is that the noise is usually stronger than the signal, making it unlikely to reach statistical significance. We should not examine the statistics in isolation. Instead, it is more meaningful to place the results in a larger context. According to our surveys (subsections II (c) and (d)), faculty and students expressed positive feelings about the API program. While all instructors highly appreciate our idea of using the API tutors to encourage EDUCO usage, and gave nearly universal praise for these tutors, they were slightly conservative in correlating our project to student success. One comment captures it well: “API cannot do it alone.” The prevailing sentiment is that external factors such as family and work responsibilities, as well as student indifference, remain the greatest obstacles to students’ academic success.
Related Projects in the Future

Together with our Grants Office and Academic Affairs Division, we will continue to monitor external funding opportunities, including NSF and the U.S. Department of Education programs. (LaGuardia has been very successful in winning FIPSE and Title V grants from the latter agency.) We will also consult with our Institutional Research Office, and evaluate the feasibility of tracking the sequential performances of this cohort of students.

Plan for Dissemination

On September 10, 2011, we will present our results and share some EDUCO implementation tips during the semi-annual Basic Skills Workshop for full-time and adjunct LaGuardia math faculty. We plan to present our findings at New York State Mathematics Association of Two-Year Colleges (NYSMATYC) conference, and possibly also Mathematical Association of America (MAA) conference. Both conferences are heavily attended by CUNY and SUNY math faculty, whose student backgrounds are similar to ours. We will also explore other publication venues (such as magazines or newsletters) to share our experience with a wider audience.
Increasing Student Success and Retention in Mathematics through Student-Centered Instruction and Collaborative Learning

Associate Professor Claire Wladis, Mathematics Department
Associate Professor Alla Morgulis, Mathematics Department
Borough of Manhattan Community College
Background and Methods

Treisman has shown that the success of minority students in prerequisite courses is strongly dependent upon their integration into a cooperative learning community, and research confirms that individuals learn Science, Technology, Engineering, and Mathematics (STEM) disciplines best by proactively exploring and engaging in the content. Because of this, this research aimed to test the effectiveness of specific collaborative and discovery-based learning techniques in intermediate algebra, one of the primary pathway courses to higher-level mathematics for STEM majors.

This study focused on exploring class structure and method of information delivery; it was changed from a traditional lecture format to a more interactive and student-centered format which integrates structured student-centered projects with a strong discovery-based learning component. This work was done in stable cooperative groups that remained the same throughout the semester. The projects themselves were highly scaffolded and aimed to have students tackle important higher-level conceptual tasks such as mathematical proofs.

Students were required to complete a database of online homework problems created by the primary investigators (PIs) and pilot instructors. Class time in pilot sections was then restructured to focus several hours per week in class discussion and the completion of discovery-based learning projects cooperative groups. As a part of the pilot section development, the PIs and pilot instructors created eight original group projects covering each major topic of the course. After using these materials for the first time in the fall semester, the PIs and pilot instructors did some extensive revision to improve them for the spring semester. The PIs and pilot instructors met approximately biweekly during each semester of the project.

Each of the six pilot instructors taught one MAT 056 pilot section and one MAT 056 control section using a traditional lecture format. The control section taught by each instructor was scheduled to meet during the same general time of day as the pilot section for that same instructor: morning, afternoon, and evening sections were matched. This structure was chosen so as to minimize any increased variation in the sample that might be contributed by instructor or time of day. The control sections and pilot sections had the same homework assignments and exams, and both sections had access to the online materials for the course.

Changes During the Implementation Phase

In fall 2010, the PIs and pilot instructors regularly revised projects, developed further ones, and discussed the issues involved with the new teaching methods in the pilot sections. Instructors encountered a number of challenges during the first semester of teaching the pilot course sections, which we tried to address before implementing the pilot courses again in the spring. For example, student absences were disruptive for multi-day group projects. To resolve this problem, group projects were redesigned to be completed within a single class session and the group size was increased from two or three to four, so that the group could still work together if a student was absent. See the appendix for examples of other changes made during the implementation process.

Because the first semester in which we used the new course structure required an adjustment period (as we revised projects and course procedures and as instructors learned to teach more effectively with the new method) we expected to see significantly better results for the spring semester than the fall. We were uncertain whether or not the new course curriculum had been effective enough in the fall to return statistically significant results despite the learning curve involved in changing
the way we taught the course. The data reflected our subjective experience in this case: most of the measures we used to compare the pilot and control sections were statistically significant in the spring, but not in the fall.

**Results**

**Equivalence of Pilot and Control Groups**

To determine how comparable the pilot and control sections would be, we collected several measures of math preparedness for students in both groups:

- Average student scores on COMPASS placement exam, taken to place students out of remedial courses—scores on this test were required to meet a certain threshold in order to place out of or exit lower level remedial courses which are prerequisites for MAT 056.
- Time elapsed since students took COMPASS placement exam (typically a good measure of time elapsed since last math class passed).
- Proportion of students exempt from taking the COMPASS placement exam, due to high scores on standardized tests such as the SAT or the Regent’s high school exit exam.
- Average student score on MAT 056 pre-test given on first day of class.

We computed the appropriate z or t statistics comparing each of these four measures in the pilot group to the control group. Here is a summary of our results:

**Table 1:** Two-tailed t-tests (z-test in the case of the proportion) comparing the Fall 2010 pilot (N=141) and control (N=143) groups

<table>
<thead>
<tr>
<th></th>
<th>z/t-score</th>
<th>p-value</th>
<th>significant ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMPASS algebra score</td>
<td>-0.63</td>
<td>0.5265</td>
<td>ns</td>
</tr>
<tr>
<td>days since COMPASS was taken</td>
<td>0.41</td>
<td>0.6849</td>
<td>ns</td>
</tr>
<tr>
<td>proportion exempt from COMPASS</td>
<td>0.61</td>
<td>0.2709</td>
<td>ns</td>
</tr>
<tr>
<td>pretest score</td>
<td>-1.26</td>
<td>0.2081</td>
<td>ns</td>
</tr>
</tbody>
</table>

*ns means nonsignificant (α=0.05)*

**Table 2:** Two-tailed t-tests (z-test in the case of the proportion) comparing the Spring 2011 pilot (N=142) and control (N=136) groups

<table>
<thead>
<tr>
<th></th>
<th>z/t-score</th>
<th>p-value</th>
<th>significant ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMPASS algebra score</td>
<td>0.41</td>
<td>0.6818</td>
<td>ns</td>
</tr>
<tr>
<td>days since COMPASS was taken</td>
<td>-0.70</td>
<td>0.4830</td>
<td>ns</td>
</tr>
<tr>
<td>proportion exempt from COMPASS</td>
<td>0.35</td>
<td>0.2709</td>
<td>ns</td>
</tr>
<tr>
<td>pretest score</td>
<td>1.71</td>
<td>0.0884</td>
<td>ns</td>
</tr>
</tbody>
</table>

*ns means nonsignificant (α=0.05)*
We can see from these data that none of these measures show statistically significant differences between the control and pilot groups, either in fall 2010 or spring 2011 (using $\alpha = 0.05$ as the significance level). These results suggest that analysis comparing the pilot and control groups in both fall and spring are unlikely to be unduly affected by confounding variables, since both pilot and control groups are statistically similar on all tested measures.

**MEASURES OF STUDENT SUCCESS AND ATTITUDES IN PILOT AND CONTROL SECTIONS**

We measured the success and attitudes of students in the pilot and control sections by comparing the following three measures:

1. We compared the proportion of students who successfully completed the course, with “success” defined as a grade of “C” or better.

2. We administered a survey designed to measure students’ attitudes towards mathematics, comparing scores from the beginning and end of the semester.

3. We compared the change in score on exam questions that were given on the first day of class and again on the final exam.

**SUCCESS RATES**

Course completion rates were collected from BMCC Institutional Research at the end of each semester. Only those students who completed the course with a “C” grade or better were considered to have completed the course successfully. (Students with grades of “INC” were not counted as having successfully completed the course, although they may have done so at a later date.)

We compared the proportion of students who successfully completed the course in the pilot and control sections using a $z$-test for the comparison of two proportions. In the fall, success rates for the pilot and control groups were not statistically significantly different, at 48.5% and 51.9%, respectively. The effect size for this difference was trivial at −0.07, suggesting that even with a larger sample size we would have been unlikely to obtain statistically significant results. In contrast, the spring pilot group did considerably better than the control group, with a success rate that was 13.1 percentage points higher than the control group (61.2% in the pilot group vs. 48.1% in the control group), and this difference was statistically significant ($\alpha = 0.05$, one-tailed), with a $p$-value of 0.0197. In addition, the effect size for this difference was a medium-sized effect at 0.26.

<table>
<thead>
<tr>
<th></th>
<th>success</th>
<th>$z$-score</th>
<th>$p$-value</th>
<th>Cohen's $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>pilot</td>
<td>48.5%</td>
<td>-0.55</td>
<td>0.2912</td>
<td>-0.07</td>
</tr>
<tr>
<td>control</td>
<td>51.9%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* statistically significant ($\alpha = 0.05$, one-tailed)
These results suggest that the pilot course structure was likely effective in improving course success rates, but that its success was largely dependent upon successful implementation, which itself required practice and training for instructors and constant revision of course materials and structure in response to experiences in the classroom.

**STUDENT ATTITUDE SURVEYS**

Students in both pilot and control sections were given surveys designed to assess attitudes towards mathematics on the first day of class and again towards the end of the semester (See Appendix II). The same survey was given both times. The survey contained twenty statements to which students were asked to reply using a five-point Likert scale, reporting the extent to which they agreed with each statement. One quarter of the items were reverse-worded in an attempt to limit yea or nay-saying bias. A principal component factor analysis on these questions revealed only one major underlying factor, so we reverse-scored the reverse-worded items and simply summed the responses, since they seemed to measure only one underlying construct.

### Table 4: Success rates for Spring 2011

<table>
<thead>
<tr>
<th></th>
<th>success</th>
<th>z-score</th>
<th>p-value</th>
<th>Cohen's d</th>
</tr>
</thead>
<tbody>
<tr>
<td>pilot</td>
<td>61.2%</td>
<td>2.06</td>
<td>0.0197*</td>
<td>0.26</td>
</tr>
<tr>
<td>control</td>
<td>48.1%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*statistically significant ($\alpha=0.05$, one tailed)

We then ran an ANCOVA with the total post-survey score as the dependent variable, the type of section (control vs. pilot) as the independent variable, and the total pre-survey score as the covariate. The pre-survey score was a highly statistically significant ($\alpha = 0.01$) predictor of a student’s post-survey score with a $p$-value of $<0.0001$ in both the fall and the spring semesters; however, a student’s section type was not ($p$-value of 0.1696 in fall and 0.7619 in spring). This suggests that the pilot course structure did not have a significant impact on student attitudes as measured by our mathematics attitude survey, at least in the short term.

### Table 5: Descriptive statistics for post-survey results Fall 2010 (0-100 points possible)

<table>
<thead>
<tr>
<th>group</th>
<th>mean</th>
<th>Std. Dev.</th>
<th>Std. Err</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>pilot (n=76)</td>
<td>41.105</td>
<td>8.860</td>
<td>1.016</td>
<td>76</td>
</tr>
<tr>
<td>control (n=79)</td>
<td>39.759</td>
<td>8.128</td>
<td>0.914</td>
<td>79</td>
</tr>
</tbody>
</table>

### Table 6: Descriptive statistics for post-survey results Spring 2011 (0-100 points possible)

<table>
<thead>
<tr>
<th>group</th>
<th>mean</th>
<th>Std. Dev.</th>
<th>Std. Err</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>pilot (n=76)</td>
<td>41.092</td>
<td>10.299</td>
<td>1.104</td>
<td>87</td>
</tr>
<tr>
<td>control (n=79)</td>
<td>42.333</td>
<td>9.164</td>
<td>1.018</td>
<td>81</td>
</tr>
</tbody>
</table>
Students in both pilot and control sections were given standard departmental final exams at the end of the semester. Ten of the questions from these exams were given as a pretest on the first day of class, either with the exact same questions or with questions in which a few of the numbers had been changed. The ten questions chosen covered each of the major topics of the course. Instructors graded these exams using a rubric developed jointly, and students were awarded partial credit, up to a maximum of 5 points per question.

Because pretest and posttest data were far from normally distributed, we could not use an ANOVA or ANCOVA method directly on pretest and posttest scores. However, we computed change scores for each student by subtracting the pretest score from the posttest score to obtain the number of points gained in score from the pretest to the posttest. These change scores were normally distributed.
We then performed a t-test with the test change scores as the dependent variable and the type of section (control vs. pilot) as the independent variable. The difference in change scores in the fall was not statistically significant, with a p-value of 0.9558; however, in the spring the pilot sections had a change score that was statistically significantly higher (α = 0.01), with a p-value of 0.0403. This was equivalent to a gain in exam score that was 6.4 percentage points higher than those students in the control sections, equivalent to about two-thirds of a letter grade.

Table 10: Descriptive statistics for exam change scores Spring 2011

<table>
<thead>
<tr>
<th>variable</th>
<th>mean</th>
<th>std dev.</th>
<th>std err</th>
<th>lower 95% CL</th>
<th>upper 95% CL</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>control (n=92)</td>
<td>23.9</td>
<td>12.8</td>
<td>1.3</td>
<td>21.3</td>
<td>26.4</td>
<td>102</td>
</tr>
<tr>
<td>pilot (n=101)</td>
<td>27.1</td>
<td>13.1</td>
<td>1.3</td>
<td>24.5</td>
<td>29.7</td>
<td>99</td>
</tr>
</tbody>
</table>

Table 11: One-tailed t-test for exam change scores for Fall 2010 testing pilot > control

<table>
<thead>
<tr>
<th>ho. diff</th>
<th>mean diff.</th>
<th>SE diff.</th>
<th>T</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>3.1</td>
<td>1.8</td>
<td>-1.71</td>
<td>191</td>
<td>0.9558</td>
</tr>
</tbody>
</table>

*statistically significant (α=0.05)*

Table 12: One-tailed t-test for exam change scores for Spring 2011 testing pilot > control

<table>
<thead>
<tr>
<th>ho. diff</th>
<th>mean diff.</th>
<th>SE diff.</th>
<th>T</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>-3.2</td>
<td>1.8</td>
<td>-1.76</td>
<td>199</td>
<td>0.0403</td>
</tr>
</tbody>
</table>

*statistically significant (α=0.05)*

As with our analysis of student success rates, these results suggest that the pilot course structure was likely effective in improving student understanding of course material, but that the success of the pilot course structure was largely dependent upon successful implementation after a period of practice, training and revision.
Conclusions

The statistical analysis for this study seems to suggest that specific collaborative learning projects used as a part of a comprehensive course structure can have a significant effect on student success. Of course, our findings must be considered in light of the study’s limitations. The teaching style in implementing a collaborative learning structure varied a great deal from one instructor to the next, and it is likely that the intervention effectiveness is dependent upon instructor. In addition, it is possible that the benefits of this intervention were not the result of collaborative learning per se, but rather of other features of the projects; a follow-up analysis of the actual nature of student interaction in pilot classes could be used to clarify this. Keeping these limitations in mind, it seems that collaborative learning can work very effectively but that there is a learning curve, for both instructors and curriculum developers. However, collaborative group work in stable base groups can lead to increases in student performance on exams of approximately two-thirds of a letter grade and about a 13 percentage point gain in successful course completion compared to standard courses using a lecture format as the primary course structure.

Future Research

We plan to track the students who participated in our study over the next several years to see if the experience using collaborative learning projects in intermediate algebra increased the rate at which students succeeded in subsequent mathematics courses, or the rate at which students chose and/or graduated with STEM majors.

Because our results suggested that an instructor’s experience with this teaching method improved the success of the method over time (as the instructor learned how to anticipate and cope with common issues and as course materials were revised), it would be interesting to study to what extent this improvement might continue in subsequent semesters. At what point is an instructor likely to be proficient enough at teaching this kind of course that significant further improvements are not likely solely based on further instructor experience in the method? How much more improvement is possible with further instructor experience and training?

This study focused on one particular kind of group project and class structure, but there are a number of variations in this course structure which could be explored. For example, the particular group projects used in this study involved students rotating through three prescribed roles as “Prover,” “Explainer” and “Checker;” however, there is no reason why this particular structure is the only successful one for collaborative group projects. We chose to set up the projects this way so that the instructions for each project would be consistent, with the hope that students would be able to spend progressively less time and effort learning how to follow the instructions and would be able to focus more time on the math itself as the semester went on. However, there are a number of other types of collaborative group projects which we would like to try out in a class structure similar to the one used in this study to see which kinds of projects are likely to be the most effective.

Another issue that came up during the implementation of this project was that it could be tricky finding the right balance between holding the groups accountable and holding individuals accountable for their learning. One of the changes we made from the fall to the spring semester was to follow each group project with an individual quiz based on the group project type questions. But to what extent
was the success of the course structure based on this change alone? Would repeating our course structure with a few different types of grading and assessment change the effectiveness of the group projects? How important is assessment type in obtaining effective results?

It would also be interesting to explore to what extent professor teaching techniques and attitudes affect the success of the collaborative learning environment. We had hoped to conduct such an analysis with our current study, but because the fall and spring semesters were so different, it didn’t make sense to pool them together, and as a result, the sample sizes just weren’t quite large enough for such an analysis to make sense. However, the group of pilot instructors who participated in this study varied widely in their level of strictness, the way in which they assigned homework, the extent to which they believed in the possible efficacy of the pilot course structure, etc. It would be interesting to see to what extent different instructor attitudes and teaching styles would affect the effectiveness of the pilot program by choosing a larger sample and by tracking more specifically how each individual pilot instructor conducts their pilot and control courses.

**External Grant Funding and Dissemination**

We are currently working on an NSF TUES grant application and a Department of Education IES research grant application with the aim of furthering the progress we’ve begun with this current project. We have two papers in the editing stage: one which analyzes the data from this project, and one which presents, in a more narrative fashion, the individual experiences which instructors had in implementing the pilot course structure and assignments in their teaching. After encountering some initial challenges in implementation, we discovered the importance of discussing practical challenges and solutions in effectively implementing cooperative group projects. We are also currently drafting a guide containing all of the group projects from our study, and all of the student and faculty instruction sheets and student worksheets, along with descriptions of the procedures and techniques that we refined along the way. When finished, we hope to be able to publish this both in hard copy and electronically. We have several conference talks planned as well to distribute the results. We plan to present our results at the 2012 Joint Mathematics Meetings in Boston, at the 2012 MAA SIGMAA on RUME conference in Portland, at the 38th Annual AMATYC Conference in Jacksonville, and at the 2011-2012 Northeast regional MAA and NYSMARTYC meetings.
References


Appendix I: Changes Made From Fall to Spring Implementation

Here are some of the conclusions that were drawn by pilot faculty during and after their experiences teaching in the fall, and the steps taken between fall and spring to address each of these issues:

**Issue:** There was concern that the group project grades could inflate student grades too much, since they were assigned for the whole group—students who did not truly understand the material might be given a higher grade simply because of high group project participation. On the other hand, simply reducing the total percentage of the grade coming from group projects was likely to reduce student motivation to take the projects seriously.

**Spring changes:** Group project grades were changed to consist of two parts: 1) the group project itself, which is a group grade and 2) a single question quiz (10-15 min) for each group project, which mimics the kinds of questions from the project, which each student will complete individually in class after the project is over. The group project grades were split evenly between these two parts, to measure both group effort and individual comprehension.

**Issue:** Group projects were sometimes too long, which made it difficult to complete the syllabus. Instructions for group projects involved some repetition from one group project to another, making the projects longer, and making it more difficult to clearly see the main points.

**Spring changes:** Several group projects were cut down to make them shorter and more manageable; each was reformatted so that there was a single clear and consistent organizational principle throughout all projects and so that the different parts (explanation, examples, student problems to be answered) were clearly differentiated; and any instructions that were repeated from project to project were removed. A single Group Project Instruction Sheet which would work for all projects was developed, photocopied onto neon colored paper, and distributed on the first day of class. A suggested week-to-week schedule was also created to help faculty pace both pilot and control sections so that all topics would be covered without rushing at the end of the semester.

**Issue:** When students were absent, it made the group work much more difficult—students ended up having to join another group halfway through the project, or a group was left without their work from the previous class when an absent classmate was the one responsible for bringing it to the next class.

**Spring changes:** Groups were chosen so that they contained four students each instead of two to three. Students were required to each have a project folder that would contain the Group Project Instruction Sheet, and which they would be required to bring to every class. Faculty were to then collect the group projects at the end of every class, and then redistribute them at the beginning of the next class, so that students don't have the problem of having a missing classmate absent with the project the next day that the group works on it in class.

**Issue:** Students were not consistently following group project instructions requiring them to rotate roles and work together to discuss answers. Students were unclear about how to write up their group projects, and were leaving out important information, or submitting it in a disorganized manner.

**Spring changes:** A Group Project Report Sheet was developed that required students to write out the names of which person completes each task, and which gave students a clear organizational structure for writing up projects. Students would then complete group projects on a Group Project Report.
Sheet, one report per group per project. Instructors also discussed the importance of their own enforcement of group project instructions—if the instructor noticed one student doing more of the work than the others, and taking over their assigned roles during the project, or students working separately within the group without discussing their answers, the instructor’s quick and consistent correction could eliminate this issue in future projects.

Appendix II: Student Survey

Each question was accompanied by the following five-point Likert scale:

*strongly agree*—*agree*—*neither agree nor disagree*—*disagree*—*strongly disagree*

1. I enjoy doing mathematics.
2. I am good at math.
3. If I get bogged down in a math problem I am confident that I can usually find my way out.
4. I feel comfortable asking questions in my classes when I don’t understand things about math.
5. I am comfortable explaining mathematical ideas to others.
6. I am going to study more math.
7. Anyone who works hard can do reasonably well at math.
8. People are either naturally good at mathematics or they are not.
9. Mathematics is intrinsically more difficult than other subjects.
10. If one way of solving a problem doesn’t work, I try another method.
11. If I can’t get the idea of a problem right away, I probably can’t get it.
12. In mathematics I can be creative and discover things for myself.
13. Understanding “why” a math problem has a particular answer is often as important as knowing what the answer is.
14. In mathematics, exploring ways to solve a problem is as important as getting the “right” answer.
15. After I’ve forgotten all the formulas, I’ll still be able to use ideas I’ve learned.
16. Mathematics is a solitary activity, done by individuals in isolation.
17. Explaining mathematics to other students and discussing mathematics with my classmates helps me to learn mathematics better.
18. When learning math, I prefer to work with others.
19. Beyond passing a required course, I don’t see the reason for learning the mathematics I am studying.
20. Mathematics is important for my chosen profession.
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