FINAL REPORT ON THE CUNY IML RESEARCH PROJECT:
Teaching Pre-Algebra and Algebra Concepts to Community College
Students through the Use of Virtual Manipulatives

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Project Background
This research project’s goal was to investigate the effectiveness – of using virtual
“manipulatives” on the learning of basic mathematical concepts by community college
students enrolled in pre-algebra and algebra remedial classes. The impact on students’
attitudes toward mathematics and their confidence in doing mathematics were also
investigated.
Manipulatives allow students to build up mental representations and acquire skills
in using and modifying these representations and synthesizing new ones (Davis 1984;
Kilpatrick 1985; Thompson, 1982). They have been described and found by many to be
the best approach to resolve the difficulties inherent in learning arithmetic and algebra
concepts and processes (Boas, 1981; Bruner, 1966; Davis, 1988; Dienes, 1971; Gningue,
Whereas hands-on manipulatives are tactile and visual, virtual manipulatives are
only visual; on the other hand, virtual manipulatives are interactive: that is, the learner
can manipulate the same objects and create the same mental representations of the objects
using the computer mouse (http://nlvm.usu.edu). In today’s technology-enriched
classrooms, it is even more appealing for college professors and college students to use
computers rather than hands-on manipulatives.
Bruner asserted that learning by discovery involved an internal reorganization of
previously known ideas and stipulated that children move through three modes or levels
of representation as they learn (Bruner, 1966). In the first or enactive level, the child
needs action on materials to understand a concept. In the second or iconic level, the child
creates mental representations of the objects but does not manipulate them directly;
rather, the concept is represented pictorially. Finally, in the third or symbolic level, the
child is strictly manipulating symbols and does not need to manipulate objects.
On the continuum from concrete to abstract representation of mathematical
concepts, virtual manipulatives fall between the enactive level and iconic level: the
student handles the squares, blocks and fraction pieces using the mouse and computer
screen. In this research, we assumed, based on our experience working with colleagues
and students, that college students might be reluctant (or offended) if asked to learn
mathematics concepts using physical manipulatives, as children do.

Research Design and Method
Two CUNY colleges were involved in this research project during the Fall, 2010,
Spring 2010 and Spring, 2011 semesters: Hostos Community College and Bronx
Community College (BCC). The two investigators (Menil from Hostos and Fuchs from
BCC) designed modules using virtual manipulatives related to the study of pre-algebra
(integers, fractions, decimals, ratios, and percents) and algebra (polynomial operations,
factoring, functions and equation solving), concepts that constitute more than 50% of the
mathematics curriculum. They taught the control groups and experimental groups in their respective campuses. Two sections of pre-algebra and two sections of algebra classes were examined. The four classes were assigned to the two investigators by their respective department chairs as part of the CUNY system of faculty teaching load. The experimental groups primarily learned the pre-algebra and algebra concepts using the technology on virtual manipulatives while the control groups learned the same concepts through the traditional lecture-style pedagogy. A one-way Analysis of variance (ANOVA) was used to investigate students’ performance on pre-algebra and algebra concepts with and without the use of virtual manipulatives. The study used the technology on virtual manipulatives as the independent variable. The students’ scores in the different assessments in both the pre-algebra and algebra were the dependent variables. Results of the Fennema-Sherman Mathematics Attitude Scales were a part of the qualitative analysis of students’ attitudes toward mathematics and confidence in doing mathematics.

This research study was quasi-experimental. The sample, represented by students in the two gateway courses, pre-algebra and algebra, was of convenience since the investigators taught these courses; however, the classes selected for intervention (technology-virtual manipulatives) and control (lecture) were randomly selected by the students who, upon registering, were not aware of this research project, let alone knowing whether they would be part of an experimental or a control group.

On the first day of classes, the students received a detailed explanation of the purpose of the project. The two investigators explained the content of the IRB consent form before they signed it. The students were given the option of not participating in the project and switching to a different section. In both colleges, all the students agreed to participate in the research project and to stay in their respective group. None of the students elected to switch sections, neither at Hostos nor at BCC.

When signing the consent forms the students in the experimental groups were given the option not to be audio recorded, not to be video recorded, or both. The investigators honored the students’ elections throughout the project.

**Data collection**

*Qualitative data collected*

- Students provided responses to a 24-question survey on attitudes toward mathematics and confidence in doing mathematics. The questionnaires were based on two subscales of a revised version of the Fennema-Sherman Mathematics Attitude Scales (Hackett & Betz, 1989). Identical questionnaires were given to the students at the beginning and end of each class. As a certain percentage of the students (approximately 20%) dropped out of the classes before conclusion, not all students who filled the questionnaires at the beginning of the class filled questionnaire at the end of the class.

- The classes were audio / video recorded, sometimes in their entirety, sometimes for only part of the class. We put more emphasis on video recording of students’ work with the virtual manipulatives in the experimental classes than on students’ work in the control groups, since teaching in the control group classes did not differ from the traditional teaching used in other sections in the two colleges. The video recording was done by the research assistants, or in the case of BCC by the
lab technician as well. Some classes were video recorded in their entirety by using one (or two) video recorders.

- The recorded events were analyzed by the investigators, who sought patterns of learning and compared the learning with virtual manipulatives to the learning in lecture-format classrooms. The use of students as co-teachers in the technology-infused classroom was documented through video recordings as well. The investigators produced several video clips illustrating the learning by the students who used virtual manipulatives.
- Several students wrote weekly reflections that provided investigators with insight into their mathematical difficulties and attitudes toward use of technology in learning mathematics. Some students also produced mathematical autobiographies that provided the investigators with insight into the students’ home culture and exposure to learning of mathematics in their elementary, middle and high school years.

Quantitative data collected in the pre-algebra classes

- Students’ responses to the initial pre-algebra test (composed by the two investigators) consisting of 25 questions covering the following topics: operations with whole numbers, operations with fractions, operations with decimals and percentages, ratios and proportions, order of operations, operations with signed numbers, and related word problems. This test was administered during the first week of the pre-algebra classes.
- Students’ responses to a pre-final test consisting of the same 25 questions that appeared in the initial test mentioned above. This test was administered during the last week of the pre-algebra classes and provided an indication of the individual student’s learning as well as the understanding of the entire class.
- Students’ scores in the achievement tests at the conclusion of the units dealing with integers; fractions and decimals; ratio and proportion; percents; uniform pre-algebra Hostos and BCC final examinations (different examinations were used by the respective mathematics department at the two colleges).
- Students’ scores in the COMPASS exit test. These data were not compiled in the production of this report, since the two colleges have different criteria for allowing students to take the COMPASS as an exit exam:
  - At Hostos only students who passed the departmental mid-term examination were allowed to participate in the COMPASS exit exam. This policy is strictly enforced. At BCC all students who participated in this project were allowed to take the COMPASS exit exam.
  - There is a difference of policy between the two colleges in what constitutes the passing grade on the COMPASS test. The requirements at Hostos are stricter than the requirements at BCC.
  - CUNY decided that effective January 2011 the COMPASS test is going to be used only as an entrance exam and for course placement purposes, but not as an exit exam for evaluating student learning.

Quantitative data collected in the algebra classes

- Students’ scores in the achievement tests at the conclusion of the units dealing with polynomial operations; factoring; functions and solving linear equations.
• Students’ scores in the uniform algebra Hostos and BCC final examinations (different examinations were used by the respective mathematics department at the two colleges).
• Students’ scores in the COMPASS exit exam. These data were not compiled for producing this report, for the reasons explained above.

In this research project, the following hypotheses were tested:
1. Due to the intervention (virtual manipulatives) significant differences in the students’ performance in pre-algebra are expected, with experimental group doing better than the control group;
2. Due to the intervention (virtual manipulatives) significant differences in the students’ performance in algebra are expected, with experimental group doing better than the control group;
3. Due to the intervention (virtual manipulatives) there will be a significant improvement in experimental group students’ confidence in their mathematical ability
4. Due to the intervention (virtual manipulatives) there will be a significant improvement in experimental group students’ attitude toward mathematics

Results and Findings

HOSTOS PRE-ALGEBRA
Table 1 shows that for the Hostos pre-algebra classes, the experimental group outperformed significantly the control group in all five assessments: Exam 1 (fractions), Exam 2 (decimals), Exam 3 (percents) Exam 4 (integers), and the Final Examination. The mean differences of the five assessments between the experimental and control groups were all significant, (p < .05 and p < .10). Please see Table 1 for findings. Overall, the significant differences were in favor of the experimental group.

<table>
<thead>
<tr>
<th>Table 1: Hostos Pre-Algebra (Fall 2010): Experimental vs. Control Means on the Five Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exam</strong></td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>Exam 1</td>
</tr>
<tr>
<td>Exam 2</td>
</tr>
<tr>
<td>Exam 3</td>
</tr>
<tr>
<td>Exam 4</td>
</tr>
<tr>
<td>Final</td>
</tr>
</tbody>
</table>
Figure 1:
Hostos Pre-Algebra (Fall 2010): Experimental vs. Control Means on the Five Assessments

![Graph showing experimental vs. control scores](image)

BCC PRE-ALGEBRA

Table 2 reveals that for the BCC pre-algebra classes, the mean scores for the control group are higher than that of the experimental group in all the five assessments: Exam1, Exam 2, Exam 3, Exam 4 and the Final Examination. Notice that Exam 4 has the smallest mean difference in favor of the control group. The graph shows significant mean differences in all the five assessments in favor of the control group, \( p < .001, p < .01, p < .05 \) and \( p < .10 \). Factors that seem to have contributed to the control group’s better performance over the experimental group include time effect (8:00 am class vs. 10:00 am); age effect (younger students vs. mature students); student preparation (inadequately prepared vs. better prepared students). See Table 2 for the significant findings in favor of the control group.

Table 2:
BCC Pre-Algebra (Fall 2010): Experimental vs. Control Means on the Five Assessments

<table>
<thead>
<tr>
<th>Assessments</th>
<th>EXPERIMENTAL</th>
<th>CONTROL</th>
<th>n</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXAM1</td>
<td>Mean 56.4211</td>
<td>SD 20.3369</td>
<td>19</td>
<td>SIG @ .001</td>
</tr>
<tr>
<td></td>
<td>EXPERIMENTAL</td>
<td>76.7500</td>
<td>11.2607</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>CONTROL</td>
<td>62.8125</td>
<td>19.7018</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>EXPERIMENTAL</td>
<td>78.8261</td>
<td>12.2053</td>
<td>23</td>
</tr>
<tr>
<td>EXAM3</td>
<td>EXPERIMENTAL</td>
<td>68.1765</td>
<td>14.1476</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>CONTROL</td>
<td>80.2632</td>
<td>9.6370</td>
<td>19</td>
</tr>
<tr>
<td>EXAM4</td>
<td>EXPERIMENTAL</td>
<td>73.5714</td>
<td>17.0687</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>CONTROL</td>
<td>82.2727</td>
<td>9.6913</td>
<td>22</td>
</tr>
<tr>
<td>FINAL</td>
<td>EXPERIMENTAL</td>
<td>65.8125</td>
<td>23.1177</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>CONTROL</td>
<td>78.7000</td>
<td>12.1400</td>
<td>20</td>
</tr>
</tbody>
</table>
Figure 2:
BCC Pre-Algebra (Fall 2010): Experimental vs. Control Means on the Five Assessments

Table 3 shows that for the Hostos algebra classes, the mean performance of the experimental group is higher in three of the four assessments: Exam 1, Exam 3 and the Final Exam. However, the mean differences in these three exams between the experimental and control were not all significant. The graph shows that the mean difference in exam3 is the only assessment that resulted to a significant finding (p < 0.05). See Table3.

Table 3:
HOSTOS Algebra (Spring 2011): Experimental vs. Control Mean Comparisons on the Four Assessments

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>n</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXAM 1</td>
<td>EXPERIMENTAL</td>
<td>65.9286</td>
<td>23.9164</td>
<td>28</td>
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<tr>
<td></td>
<td>CONTROL</td>
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<td>30.0166</td>
<td>28</td>
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<td>EXAM 2</td>
<td>EXPERIMENTAL</td>
<td>69.3571</td>
<td>28.5362</td>
<td>28</td>
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<tr>
<td></td>
<td>CONTROL</td>
<td>79.8333</td>
<td>23.0551</td>
<td>24</td>
</tr>
<tr>
<td>EXAM 3</td>
<td>EXPERIMENTAL</td>
<td>72.8800</td>
<td>20.2471</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>CONTROL</td>
<td>55.6190</td>
<td>26.5207</td>
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</tr>
<tr>
<td>FINAL</td>
<td>EXPERIMENTAL</td>
<td>52.1600</td>
<td>21.7269</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>CONTROL</td>
<td>48.1053</td>
<td>24.1452</td>
<td>19</td>
</tr>
</tbody>
</table>
Figure 3:
HOSTOS Algebra (Spring 2011): Experimental vs. Control Mean Comparisons on the Four Assessments

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BCC ALGEBRA
Table 4 shows that for the BCC algebra classes, the experimental group had a higher mean performance than the control group in all the four assessments. However, the mean differences in these four assessments were not all significant. The graph shows that the mean difference in Exam 2 was the only assessment that was significant (p < 0.05). See Table 4.

Table 4:
BCC Algebra (Spring 2010 and Spring 2011): Experimental vs. Control Mean Comparisons on the Four Assessments

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>n</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXAM1</td>
<td>EXPERIMENTAL</td>
<td>65.0000</td>
<td>27.5780</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>CONTROL</td>
<td>57.6800</td>
<td>21.4218</td>
<td>25</td>
</tr>
<tr>
<td>EXAM2</td>
<td>EXPERIMENTAL</td>
<td>71.3684</td>
<td>25.2019</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>CONTROL</td>
<td>56.0909</td>
<td>20.1823</td>
<td>22</td>
</tr>
<tr>
<td>EXAM3</td>
<td>EXPERIMENTAL</td>
<td>69.7895</td>
<td>25.8963</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>CONTROL</td>
<td>60.5500</td>
<td>19.4733</td>
<td>20</td>
</tr>
<tr>
<td>FINAL</td>
<td>EXPERIMENTAL</td>
<td>67.3500</td>
<td>24.6177</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>CONTROL</td>
<td>60.0000</td>
<td>18.9816</td>
<td>21</td>
</tr>
</tbody>
</table>

Figure 4:
BCC Algebra (Spring 2010 and Spring 2011): Experimental vs. Control Mean Comparisons on the Four Assessments
Interpretation of Results

Pre-algebra
At Hostos pre-algebra classes, the experimental group outperformed significantly the control group in all the five assessments, thereby confirming one of our initial hypotheses \(p < .05\) and \(p < .10\). By contrast, BCC pre-algebra control group performed significantly better than the experimental group in all the five assessments \(p < .001, p < .01, p < .05, p < .10\). There were several reasons for the weaker performance of the students in the experimental group at BCC:
- The control group students arrived to college better prepared in mathematics than the experimental group students; this was well documented by the initial placement COMPASS test results. By sheer chance, the two groups differed significantly in their preparation for college work.
- The experimental group started at 8:00am; many students arrived late to class or lab. Some single mothers had difficulty arranging for childcare in the early morning hours. Several single men worked night jobs and arrived to school late.
- The control group students displayed a more mature attitude toward learning mathematics than the experimental students.

Algebra
- At Hostos algebra classes, the mean scores of the experimental group were higher than the control group in three of the four assessments. However, the mean differences in these three assessments were not all significant. It was only significant in exam3, \(p < .05\), thereby confirming one of our initial hypotheses. In the other campus at the BCC algebra classes, the mean scores of the experimental group were higher than the control group in all the four assessments. However, the mean differences in these four assessments were not all significant. It was only significant in exam2, \(p < .05\), thereby confirming one of our initial hypotheses, that due to the intervention (virtual manipulatives) there will be a significant difference in students’ performance in these two gateway courses: pre-algebra and algebra.

Students' Attitudes and Confidence
From the pre-survey to the post-survey, the overall response mean in attitude and confidence significantly changed for only one of the eight groups: the BCC algebra control group \(p < .05\). When all questions were studied individually however, an increase in the mean responses was noted for all groups for each one of the questions. For two of the “confidence” questions, “I am no good at math,” and “Math has always been my worst subject,” a significant change \(p < .05\) was found in both experimental pre-algebra classes at BCC and Hostos. This result leads us to believe that the confidence in doing mathematics level of students in the Pre-Algebra Experimental groups’ grew more significantly after using the virtual manipulatives than the confidence level of students in the pre-algebra control groups’ at BCC and Hostos after being taught traditionally.

For the algebra groups, no significant changes were noted from the pre- to post-surveys on any of the attitudes and confidence indicators. The control groups at both
BCC and Hostos displayed however, a more positive attitude and had a higher confidence level than any of the other groups at the beginning of the algebra courses. Since algebra students have already passed pre-algebra, they perhaps believe more in their ability to do math than pre-algebra students.

**Effects of Technology**

The infusion of technology in the experimental groups promoted group work and facilitated co-teaching. In these classes students volunteered to be assistant teachers and to help other students. Altogether, these classes were more student-centered. Students learned and practiced many concepts on their own, not needing the teacher to come verify their answers. In the tedious work of subtraction of integers, for instance, the concept of zero-pair is essential to understanding the meaning of subtraction. The technology enabled students to try different combinations and strategies on what type of zero-pair to add, and check the results immediately to validate their answers. In instances when the technology was not available, students struggled to understand the concepts taught while waiting for better teacher explanation. The learning with technology allowed the more conscientious students to repeat the lessons at home, on their computer, until they mastered the concepts. The computer software provided the students with many new practice exercises and instant feedback.

**Conclusions and Recommendations**

We obtained mixed results as far as the quantitative data were concerned. The qualitative data, however, confirmed our hypotheses that virtual manipulatives were useful for students learning basic mathematics concepts. Students expressed this usefulness clearly in their reflections, in face-to-face interviews and through their answers in the questionnaires on attitudes toward mathematics and confidence in doing mathematics.

The experimental group students overcame their initial mathematics misconceptions with less difficulty than the students in the control group. They found the classes with virtual manipulatives more exciting than traditionally taught mathematics classes. The virtual manipulatives appear to be more useful in teaching pre-algebra remedial course than an algebra remedial course. The reasons are as follows:

- The modules for pre-algebra cover most topics: fractions, decimals, percentages, ratios and proportions, operations with integers and solving linear equations;
- The modules for algebra do not cover inequalities, factoring of polynomials and solving quadratic equations;
- The algebra curricula at both Hostos and BCC are accelerated. Teaching a topic with virtual manipulatives takes longer than teaching the same topic in a traditional lecture-style class. Consequently, learning of algebra with virtual manipulatives require more hours than a typical one-semester class;
- By its nature, algebra is more abstract than pre-algebra. Students learning algebra concepts are not helped as much as students learning pre-algebra with virtual manipulatives.
Future studies are needed to determine the longer-term utility of learning remedial mathematics with virtual manipulatives. We could hypothesize that students who learned and mastered mathematics concepts with virtual manipulatives will retain the concepts longer. Should virtual manipulatives be an integral part of teaching remedial mathematics in CUNY? To answer this question we have to consider the investment in technology, the training of faculty and the willingness of the faculty to embrace technology in teaching. In order to maximize the effectiveness of using virtual manipulatives, we recommend the following:

1. Provide an intensive two-week workshop series using virtual manipulatives to those students who failed the pre-algebra COMPASS placement test and those students who are multiple repeaters of the remedial pre-algebra course;
2. Provide training in the use of technology to faculty members who will be conducting those workshops.

We plan to disseminate our findings in seminars relating to developmental mathematics. We also plan to publish our findings in a peer-reviewed journal.

**References**


Gningue, S. M. (2000). *The use of manipulatives in middle school algebra*: (Committee on Prospering in the Global Economy of the 21st Century, 2006). This conclusion is consistent with the convincing evidence provided by the National Academies in

